Low Rank Representation on Product Grassmann Manifolds for Multi-view Subspace Clustering

Jipeng Guo[1], Yanfeng Sun[1], Junbin Gao[2], Yongli Hu[1] and Baocai Yin[1]

[1]Beijing University of Technology, China
[2]The University of Sydney, Australia

Contribution

- propose the Low-Rank Representation (LRR) model with Matrix Factorization on Product Grassmann manifolds (PG-MFLRR), which is a multi-view manifold clustering method;
- PG-MFLRR captures the true low rank structure of data representation. And, factorization strategy provides nearly unbiased relaxations of the rank function. They can help us to achieve the higher clustering accuracy and save the time consumption in order to have a wide range of applications;
- The existence of optimal solution for the non-convex optimization problem is proved. Furthermore, an effective optimization algorithm is developed; and
- Extensive experiments are conducted on four multi-source video datasets, which demonstrate the effectiveness and competitiveness of the proposed method.

BackGround: Product Grassmann Manifolds

Grassmann manifolds [1], denoted by $\mathcal{G}(p,d)$, is the space of all p-dimensional linear subspaces of $\mathbb{R}^d$ for $0 \le p \le d$. Grassmann manifolds can be embedded into the space of symmetric matrices $\mathrm{Sym}(d)$ as

$$\Pi : \mathcal{G}(p,d) \to \mathrm{Sym}(d), \quad \Pi(X) = XX^T. \tag{1}$$

hence it is reasonable to replace the distance on Grassmann manifolds with the following distance defined on the symmetric matrix space,

$$d_G^2(X,Y) = \frac{1}{2}\|\Pi(X) - \Pi(Y)\|_F^2. \tag{2}$$

Given V Grassmann manifolds with dimensions $p_1, \cdots, p_V$ respectively, the **Product Grassmann manifolds (PGM)** (denoted by $\mathcal{PG}_{d:p_1,\cdots,p_V}$) is defined as $\mathcal{G}(p_1,d) \times \cdots \times \mathcal{G}(p_V,d)$. Then, a point embedded in PGM is a set of Grassmann points, denoted by $[X] = \{X^1, \cdots, X^V\}$ such that $X^v \in \mathcal{G}(p_v,d)$, $v = 1, \cdots, V$. A valid distance on PGM can be induced from the individual distance (2) on each Grassmann manifold as follows,

$$d_{\mathcal{PG}}^2([X],[Y]) = \sum_{v=1}^{V} d_G^2(X^v, Y^v). \tag{3}$$

Related Works

Spectral Clustering (SC) is used as the framework for subspace clustering. The main challenge by using SC is to define a "good" affinity matrix (or graph) $Z \in \mathbb{R}^{n\times n}$. To explore the global structure, LRR enforce the low rank constraint on the self-expression coefficients matrix of data. Given $Y = [y_1, y_2, \cdots, y_N] \in \mathbb{R}^{d\times N}$ denotes a set of samples collected from multiple independent subspaces, LRR [3] model is formulated as

$$\begin{aligned} \min_{Z,E} \quad & \mathrm{rank}(Z) + \lambda\|E\|_F^2, \\ \text{s.t.} \quad & Y = YZ + E, \end{aligned} \tag{4}$$

As a common precessing, the low rank term is replaced by nuclear norm as the effective approximations,

$$\begin{aligned} \min_{Z,E} \quad & \|Z\|_* + \lambda\|E\|_F^2, \\ \text{s.t.} \quad & Y = YZ + E. \end{aligned} \tag{5}$$

Once obtaining the coefficient matrix Z, spectral clustering algorithm can be performed to receive the final results.

Main Reference

[1] B. Wang, Y. Hu, J. Gao, Y. Sun and B. Yin. Low Rank Representation on Grassmann Manifolds. In ACCV 2014.
[2] B. Wang, Y. Hu, J. Gao, Y. Sun and B. Yin, Product grassman manifold representation and its lrr models. In AAAI, 2016.
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LRR with Matrix Factorization on Product Grassmann Manifolds

To handle multi-view data from multi-dimensional subspace, we consider the generalization of model (5) onto Product Grassmann manifolds. $\mathcal{X} = \{[X_1], [X_2], \cdots, [X_n]\}$ be a set of given PGM samples, where $[X_i] = \{X_i^1, \cdots, X_i^V\} \in \mathcal{PG}_{d:p_1,\cdots,p_V}$ with the basic matrix $X_i^v \in \mathcal{G}(p_v, m)$. Mathematically, Low Rank Representation on Product Grassmann Manifolds (PG-LRR) [2] can be formulated as:

$$\min_{Z} \sum_{i=1}^{n} \|[X_i] \ominus (\biguplus_{j=1}^{n} z_{ij} \odot [X_j])\|_{\mathcal{PG}} + \lambda\|Z\|_*, \tag{6}$$

where abstract symbols $\ominus$, $\biguplus_{j=1}^{n}$ and $\odot$ denote the "linear" operations to be defined on manifolds, i.e., addition, subtraction and scalar multiplication. $\|[X_i] \ominus (\biguplus_{j=1}^{n} z_{ij} \odot [X_j])\|_{\mathcal{PG}}$ with operator $\ominus$ representing the product manifold distance between $[X_i]$ and its reconstruction $\biguplus_{j=1}^{n} z_{ij} \odot [X_j]$.

Motivated by the matrix factorization, we assume that the representation matrix Z can be decomposed into tri-matrix multiplication, this is $Z = UMV^T$, where $U \in \mathbb{R}^{n\times k}$, $M \in \mathbb{R}^{k\times k}$ and $V \in \mathbb{R}^{n\times k}$, k is the given upper bound of the true rank. Two reasonable constraints $U^TU = I_k$ and $V^TV = I_k$ are introduced to ensure the stability of the solutions. Thus, $\|Z\|_* = \|UMV^T\|_* = \|M\|_*$. The **LRR with Matrix Factorization on Product Grassmann Manifolds (PG-MFLRR)** is formulated as:

$$\begin{aligned} \min_{U,M,V} \quad & \sum_{i=1}^{n} \|[X_i] \ominus (\biguplus_{j=1}^{n} z_{ij} \odot [X_j])\|_{\mathcal{PG}} + \lambda\|M\|_*, \\ \text{s.t.} \quad & Z = UMV^T, U^TU = V^TV = I_k, \end{aligned} \tag{7}$$

where $Z = \{z_{ij}\}_{i,j=1}^{n} \in \mathbb{R}^{n\times n}$ is the coefficient representation matrix.

Main Experiments

Method	Metrics	ACT4[2]	NUCLA	IXMAS	DTHC
$SCGSM_{best}$	ACC	0.4405	0.2823	0.4189	0.7189
	NMI	0.5065	0.2115	0.4507	0.5499
	F-score	0.1365	0.2827	0.0922	0.6531
$G\text{-}LRR_{best}$	ACC	0.4541	0.2904	0.4196	0.7802
	NMI	0.5421	0.2202	0.4669	0.6870
	F-score	0.1296	0.2976	0.4187	0.7059
SwMC	ACC	0.1139	0.2823	0.4071	0.5275
	NMI	0.0593	0.2481	0.5290	0.2649
	F-score	0.1281	0.2990	0.4570	0.5030
MLAN	ACC	0.1397	0.2703	0.2684	0.5275
	NMI	0.0821	0.2409	0.3546	0.2649
	F-score	0.1454	0.2815	0.3269	0.5030
MVGL	ACC	0.1269	0.2775	0.4041	0.7802
	NMI	0.0492	0.2024	0.4831	0.6870
	F-score	0.1298	0.2932	0.4289	0.7913
MCGC	ACC	0.1429	0.2679	0.4071	0.9396
	NMI	0.0649	0.1972	0.4448	0.8436
	F-score	0.1445	0.2710	0.4295	0.9393
SM^2SC	ACC	0.1463	0.1100	0.3835	0.7637
	NMI	0.0628	0.0042	0.4095	0.5275
	F-score	0.0782	0.1798	0.2808	0.6399
LCRSR	ACC	0.3766	0.2700	0.3890	0.8381
	NMI	0.2397	0.2078	0.3740	0.7237
	F-score	0.2217	0.0641	0.3889	0.7734
PG-LRR	ACC	0.4957	0.2969	0.4240	0.8022
	NMI	0.6250	0.3525	0.4773	0.6075
	F-score	0.5102	0.3017	0.4268	0.8014
PG-MFLRR	ACC	**0.5098**	**0.3560**	**0.4945**	**1.0000**
	NMI	**0.6421**	**0.3681**	**0.5014**	**1.0000**
	F-score	**0.5317**	**0.3978**	**0.4912**	**1.0000**

Table 1:Clustering results on different multi-view video databases.

guojipeng@emails.bjut.edu.cn