Embedding shared low-rank and feature correlation for multi-view data analysis

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Introduction

■ Motivation

- Conventional multi-view subspace learning methods can not directly characterize the structure information and the consensus of multi-view data
- These methods cannot deal with noisy data well

■ Contributions

- We propose a shared double nuclear norm-based low rank model in embedding subspace to uncover the globally shared structure information among multiple feature data, and utilize a sparse term to fit the error, which makes ESLRFC robust to noise.
- We develop an enhanced correlation analysis to simultaneously exploiting the correlations among multiple feature sets and removing the redundancy of each view.
- We develop an unify optimization problem, which can be effectively solved via an alternating minimization algorithm.

Related work

-Low rank representation

$$\min_{\mathrm{H,O}} \|\mathrm{H}\|_* + \eta\|\mathrm{O}\|_{2,1}, s.t.\ \mathrm{A} = \mathrm{AH} + \mathrm{O}$$

A data matrix, H representative matrix, O error component.

-Correlation-based multi-view subspace learning

$$\max_{\mathrm{P}_1,\cdots,\mathrm{P}_m} \sum_i \sum_{j,i\neq j} tr(\mathrm{P}_i^T \mathrm{C}_{ij} \mathrm{P}_j), s.t.\ \mathrm{P}_i^T \mathrm{C}_{ii} \mathrm{P}_i = \mathrm{I}, i = 1, \cdots, m,$$

$$\mathrm{C}_{ii} = \mathrm{X}_i \mathrm{X}_i^T \quad \mathrm{C}_{ij} = \mathrm{X}_i \mathrm{X}_j^T (i \neq j)$$

Our approach

■ Embedding shared low-rank and feature correlation

$$\min_{\substack{\mathrm{U,V}\\ \mathrm{Z,P}_i,\mathrm{E}_i}} \|\mathrm{U}\|_* + \|\mathrm{V}\|_* + \lambda \sum_i \|\mathrm{E}_i\|_{2,1}$$
$$- \alpha \sum_i \sum_{j,i\neq j} tr(\mathrm{P}_i^T \mathrm{C}_{ij} \mathrm{P}_j) + \beta \sum_i \|\mathrm{P}_i\|_{2,p}^p$$
$$s.t.\ \mathrm{P}_i^T \mathrm{X}_i = \mathrm{P}_i^T \mathrm{X}_i \mathrm{Z} + \mathrm{E}_i, \mathrm{Z} = \mathrm{UV}, \mathrm{P}_i^T \mathrm{P}_i = \mathrm{I}$$
$$i = 1, 2, \cdots, m.$$

- Multiple views have common representation matrix
- Double nuclear norm is used to construct more accurate low-rank representation matrix.
- Correlation analysis reveal the correlation of multi-view data
- $\ell_{2,p}$ norm on projection matrix is used to select a subset with representative features from the multi-view data
- A unify framework, which explores the structure and correlation, is develop for seeking the robust projection
- ADMM algorithm is adopted to solve the optimization problem

Experimental Results

■ Results on three datasets

Method	Yale			eNTERFACE'05		
	l=3	*l*=4	*l*=5	*l*=60	*l*=80	*l*=100
$\mathrm{PCA}_{\mathrm{FC}}$	69.33±3.46	72.86±3.34	75.00±4.20	58.70±1.48	65.80±2.40	71.87±1.07
$\mathrm{LRE}_{\mathrm{FC}}$	69.92±2.31	74.10±3.93	77.89±3.37	58.75±1.43	65.90±2.37	71.90±1.10
MCCA	70.75±3.67	73.33±3.10	77.89±3.37	53.21±1.93	61.35±2.30	67.69±2.74
SCCA	67.50±3.83	70.38±2.11	72.78±3.79	56.33±1.97	63.16±2.25	69.03±2.03
PLS	70.58±4.30	72.86±3.15	75.00±2.58	53.90±1.34	60.48±2.65	64.95±2.90
MDcR	73.00±2.81	76.67±3.68	81.78±3.60	63.30±1.58	70.57±2.18	76.38±2.68
ESLRFC	**73.75±3.50**	**77.71±3.94**	**82.89±3.40**	**63.08±1.71**	**71.65±2.60**	**77.95±2.19**
Method	COIL-20			COIL-20 with noise		
	l=5	*l*=10	*l*=15	20% noise	30% noise	40% noise
$\mathrm{PCA}_{\mathrm{FC}}$	84.78±1.74	90.59±0.96	95.32±0.59	92.98±0.68	90.04±0.56	84.54±1.46
$\mathrm{LRE}_{\mathrm{FC}}$	84.96±1.41	91.31±1.06	95.38±0.79	93.71±0.63	92.61±0.64	90.72±0.91
MCCA	83.62±1.57	91.06±1.14	94.79±0.86	92.90±0.70	90.08±0.85	83.80±1.17
SCCA	83.75±1.74	90.67±0.98	95.21±0.57	89.97±0.83	84.19±0.80	78.97±1.59
PLS	85.01±1.44	91.23±1.01	95.26±0.79	92.49±0.75	88.61±0.92	84.08±0.69
MDcR	85.23±1.61	91.42±1.22	95.51±0.66	94.10±0.56	92.62±0.52	89.85±0.78
ESLRFC	**85.78±1.93**	**92.19±0.97**	**95.88±0.73**	**94.48±0.90**	**93.14±0.81**	**90.86±1.04**

■ Model analysis

Method	Yale	eNTERFACE'05	COIL-20
ESLRFC_λ	82.11± 2.84	76.13±2.26	94.87±0.66
ESLRFCα	81.67±3.15	68.85±0.84	95.34±0.65
ESLRFC_β	81.56±2.36	74.36±1.94	94.59±0.72
ESLRFC	**82.89±3.40**	**77.95±2.19**	**95.88±0.73**