DCT/IDCT Filter Design for Ultrasound Image Filtering

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DCT/IDCT Filter Formulation

- **Theorem:** A discrete transformation of a discrete signal, \( f(n) \) of length \( N \), over a kernel function of \( g(n,k) \) can be derived by the discrete convolution of the kernel and the flipped signal which is evaluated at \( N - 1 \).

\[
x^F(n) * h_k(n) \bigg|_{n=N-1} \Rightarrow H_k(z) = \frac{\alpha_k (1-z^{-1})}{1-2z^{-1}\cos \omega_k z^{-2}} \text{ where } \alpha_k = \cos \left( \frac{\omega_k}{2} \right).
\]

- \( X_k = c(k) \sum_{n=0}^{N-1} x(n) \cos \left[ \frac{\pi}{N} (n + \frac{1}{2}) k \right] = c(k) \left\{ x^F(n) * h_k(n) \bigg|_{n=N-1} \right\} \)

Computational Time

Number of multiplication and addition operations for computation of DCT coefficients based on three different methods for all fetus ultrasound test images with size 400 × 400.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Fast algorithm [12]</th>
<th>Proposed algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>590</td>
<td>582</td>
</tr>
<tr>
<td>Addition</td>
<td>2450</td>
<td>182</td>
</tr>
</tbody>
</table>

Experiments

- DCT-Wiener filtering

\[
\hat{H}_W(k_1, k_2) = \frac{\hat{P}_x(k_1, k_2)}{\hat{P}_x(k_1, k_2) + \lambda(k_1, k_2)\sigma^2}
\]

\( \hat{H}_W(k_1, k_2) \) is an estimate of the frequency response of the Wiener filter. \( \hat{P}_x(k_1, k_2) \) is power spectral density estimates of the noise-free image and \( \sigma^2 \) is noise variance since \( \lambda(k_1, k_2) \) is proportional to the image size.

- DCT Filtering results for the real fetal ultrasound images

- Image reconstruction of ultrasound fetus images

Error, similarity and quality metrics:

- **SNIRE** is the Statistical-Normalization Image Reconstruction Error
- **BRISQUE** is Blind/Referenceless Image Spatial Quality Evaluator
- **SSIM** is the Structural Similarity Index Measure

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