DCT/IDCT Filter Design for Ultrasound Image Filtering

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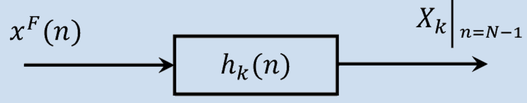
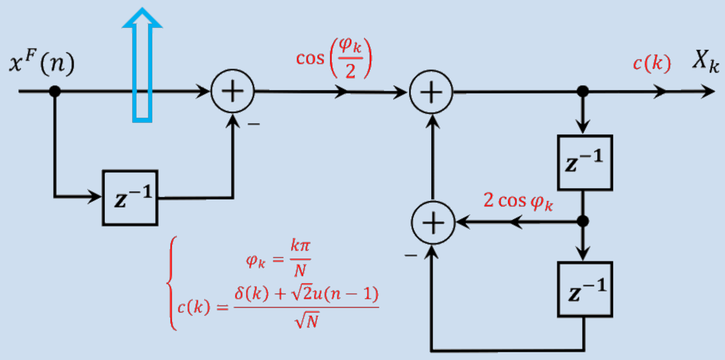
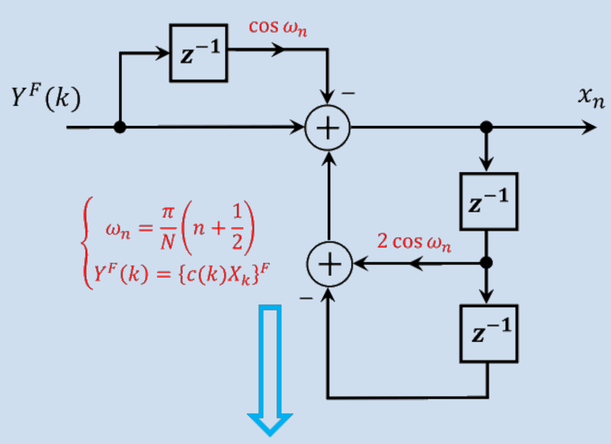
DCT/IDCT Filter Formulation

- **Theorem:** A discrete transformation of a discrete signal, $f(n)$ of length N, over a kernel function of $g(n,k)$ can be derived by the discrete convolution of the kernel and the flipped signal which is evaluated at $N-1$.



- $X_k = c(k)\sum_{n=0}^{N-1} x(n)\cos\left[\frac{\pi}{N}\left(n+\frac{1}{2}\right)k\right] = c(k)\left\{x^F(n) * h_k(n)\Big|_{n=N-1}\right\} \Rightarrow H_k(z) = \frac{\alpha_k\left(1-z^{-1}\right)}{1-2z^{-1}\cos\varphi_k+z^{-2}}$ where $\alpha_k = \cos\left(\frac{\varphi_k}{2}\right)$.



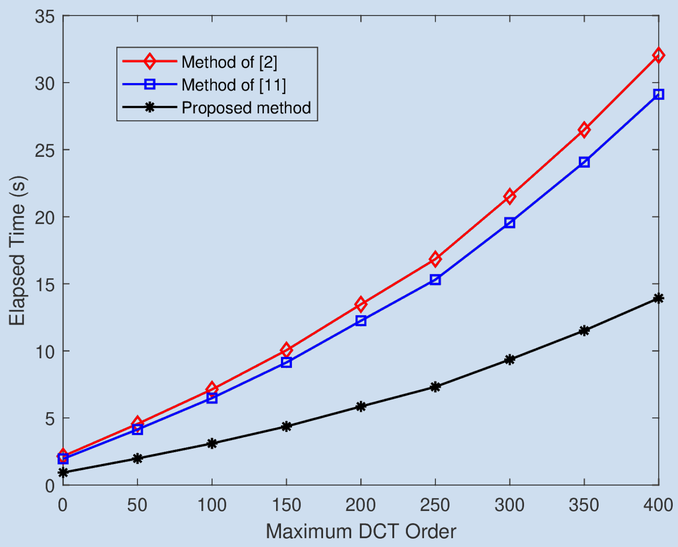


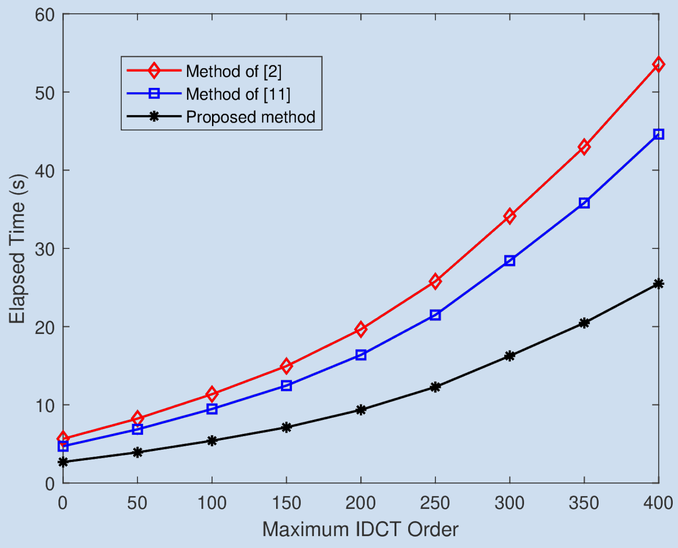
- $x(n) = \sum_{n=0}^{N-1} c(k)X_k\cos\left[\frac{\pi}{N}\left(n+\frac{1}{2}\right)k\right] = Y^F(k) * h_n(k)\Big|_{k=N-1} \Rightarrow H_n(z) = \frac{1-z^{-1}\cos\omega_n}{1-2z^{-1}\cos\omega_n+z^{-2}}$ where $\omega_n = \frac{\pi}{N}\left(n+\frac{1}{2}\right)$.

Computational Time

Number of multiplication and addition operations for computation of DCT coefficients based on three different methods for all fetus ultrasound test images with size 400×400.

Operation	Fast algorithms [2]	Fast algorithms [11]	Proposed algorithm
Multiplication	560	245	**162**
Addition	2450	N/A	**520**





Experiments

- DCT-Wiener filtering

$$\widehat{\mathbf{H}}_{\mathbf{W}}(\mathbf{k_1},\mathbf{k_2}) = \frac{\widehat{\mathbf{P}}_{\mathbf{x}}(\mathbf{k_1},\mathbf{k_2})}{\widehat{\mathbf{P}}_{\mathbf{x}}(\mathbf{k_1},\mathbf{k_2}) + \lambda(\mathbf{k_1},\mathbf{k_2})\sigma^{\mathbf{2}}}$$

$\widehat{H}_W(k_1,k_2)$ is an estimate of the frequency response of the Wiener filter, $\widehat{P}_x(k_1,k_2)$ is power spectral density estimates of the noise-free image and σ^2 is noise variance since $\lambda(k_1,k_2)$ is proportional to the image size.

- DCT Filtering results for the real fetal ultrasound images

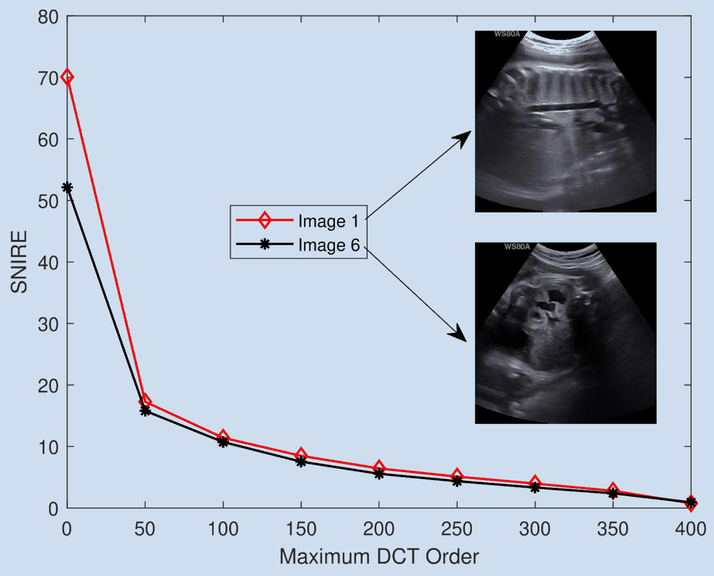
	Original image	DCT-based denoised image (proposed method)			Wiener filter denoised image
Normal Fetus					
	Threshold level (β)	0.8	0.5	0.1	N/A
	SNIRE	20.6797	17.0367	**6.7440**	**10.1939**
	BRISQUE	53.0698	50.2022	**45.4849**	**48.667**
	Threshold level (β)	0.8	0.5	0.1	N/A
	SNIRE	19.8847	16.6770	**6.6088**	**9.4221**
	BRISQUE	59.2062	58.5306	**44.5435**	**49.2863**
Fetal Cystic Hygroma					
	Threshold level (β)	0.8	0.5	0.1	N/A
	SNIRE	16.5777	13.5651	**4.5613**	**8.5121**
	BRISQUE	59.9236	59.5926	**54.4380**	**55.4829**
	Threshold level (β)	0.8	0.5	0.1	N/A
	SNIRE	15.9925	13.1133	**4.3818**	**7.3997**
	BRISQUE	54.5599	53.8146	**52.1289**	**57.1425**
Fetal Hydronephrosis					
	Threshold level (β)	0.8	0.5	0.1	N/A
	SNIRE	19.2214	15.2223	**6.3865**	**8.5244**
	BRISQUE	59.6799	53.8941	**49.1410**	**56.6273**
	Threshold level (β)	0.8	0.5	0.1	N/A
	SNIRE	19.5019	15.4862	**6.1922**	**8.4277**
	BRISQUE	66.3734	63.0046	**45.1998**	**50.2517**

- Image reconstruction of ultrasound fetus images

	Original image	Reconstructed images based on the proposed IDCT filter				
Normal Fetus						
	Max. Order	50	100	200	300	400
	SNIRE	17.2746	11.4128	6.43499	3.97367	0.765523
	SSIM	0.967772	0.986241	0.995663	0.998343	0.999935
	Max. Order	50	100	200	300	400
	SNIRE	15.5357	11.0797	6.6417	4.16901	1.19042
	SSIM	0.980865	0.990384	0.996564	0.998643	0.999883
Fetal Cystic Hygroma						
	Max. Order	50	100	200	300	400
	SNIRE	12.2475	5.42537	2.4452	1.43111	0.394929
	SSIM	0.983393	0.996762	0.999324	0.999761	0.999988
	Max. Order	50	100	200	300	400
	SNIRE	10.6938	5.50489	2.42483	1.29721	0.311768
	SSIM	0.988511	0.996958	0.999395	0.999821	0.999994
Fetal Hydronephrosis						
	Max. Order	50	100	200	300	400
	SNIRE	16.1456	10.6017	5.34303	3.34015	1.5707
	SSIM	0.949098	0.9787739	0.994671	0.997909	0.999525
	Max. Order	50	100	200	300	400
	SNIRE	15.7917	10.7213	5.55523	3.33236	0.920456
	SSIM	0.954312	0.979496	0.994569	0.998042	0.999844

- **Error, similarity and quality metrics:**

  - SNIRE is the Statistical-Normalization Image Reconstruction Error
  - BRISQUE is Blind/Referenceless Image Spatial Quality Evaluator
  - SSIM is the Structural Similarity Index Measure



Acknowledgements & Ref.

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[2] Che-Hong Chen *et al.*, IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 51, no. 10, pp. 2017–2030, Oct 2004.

[11] S. Tsai and S.-M. Yang, Mathematical Problems in Engineering, vol. 2017, 2017.