3D Pots Configuration System by Optimizing over Geometric Constraints

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## Introduction

- Potteries are frequently excavated artifacts, but they are usually found in broken fragments.
- Manual reassembly is exhaustive and inaccurate.
- There have been efforts to virtually assemble the 3D scans of fragments, but a practical and efficient method for large-scale implementation is yet to be established.
- We propose a simple yet effective algorithm to reassemble broken pottery pieces.


## Proposed Method

1. Feature Extraction

- From each scanned fragment, extract sampled break line.

Scanned 3D Mesh(Left) and Sampled Break Line(Right) of a Pottery Fragment

- Extract feature vector of each sampled point $p$, $F(p)$.
- Build a Matching-Score Matrix $\boldsymbol{M}$ such that point pairs with similar features have score approximately 1 and others 0 .

2. Optimization

- Binary optimization problem:
$\underset{\text { Similar to match score }}{\arg \min } \xlongequal[\underbrace{}_{i, j}]{\|X-M\|_{F}^{2}}+\sum_{\text {Continuity }} s_{i} s_{j}\left\|X_{i j}-X_{(i+1)(j+1)}\right\|_{2}^{2}$
Continuity
subject to $X^{\top} \mathbf{1} \leq 1, X 1 \leq 1$, Bijection
- Find a binary matrix $X$ where matched pairs of points on the break lines are encoded as 1 and other elements are 0 .
- Geometric constraints
$\checkmark$ Bijection : Each point will be matched to at most one other point. (At most one element of 1 in each row and column, and other elements should be 0 .)
$\checkmark$ Continuity : Matched points form a continuous segment on the break line. (The segment creates diagonal 1's.)
- Binary constraints are relaxed to $X^{T} \mathbf{1} \leq \mathbf{1}, X \mathbf{1} \leq$ 1, $0 \leq X$. -> Convex Quadratic Programming


## Experiment Results

- Result on Real Pottery Data

| Color |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Match detected | $x$ | 0 | 0 | 0 |
| Correct location | - | $x$ | 0 | 0 |
| Wrong location | - | 0 | 0 | $x$ | $\checkmark$ Without optimization, the feature matching scores result in hundreds of false-positives. $\checkmark$ The potteries are reassembled by attaching break lines that correspond to the peaked diagonal sequences of the optimization output $X$.



Reassembly of each pottery : 3 false-positives
$\checkmark$ Hierarchical extension : Start with a large sampling interval and iteratively solve the optimization problem, decreasing the sampling interval after each iteration.


Joint reassembly using the hierarchical extension : 10 false positives

- Result on Synthetic Data


An Exemplar
Binary Matrix
Satisfying
Bijection and Continuity

0.6


The original diagonals are recovered after our optimization for noise of standard deviation up to 0.6.

