Fixed Simplex Coordinates for Angular Margin Loss in CapsNet

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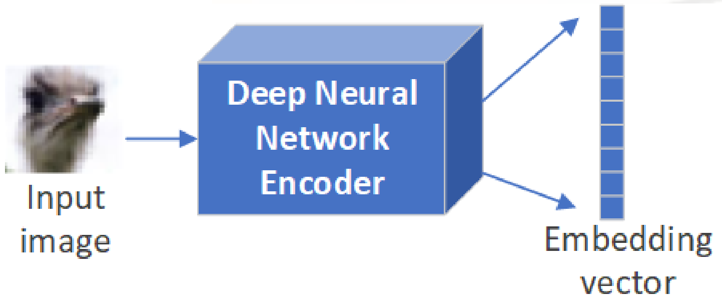
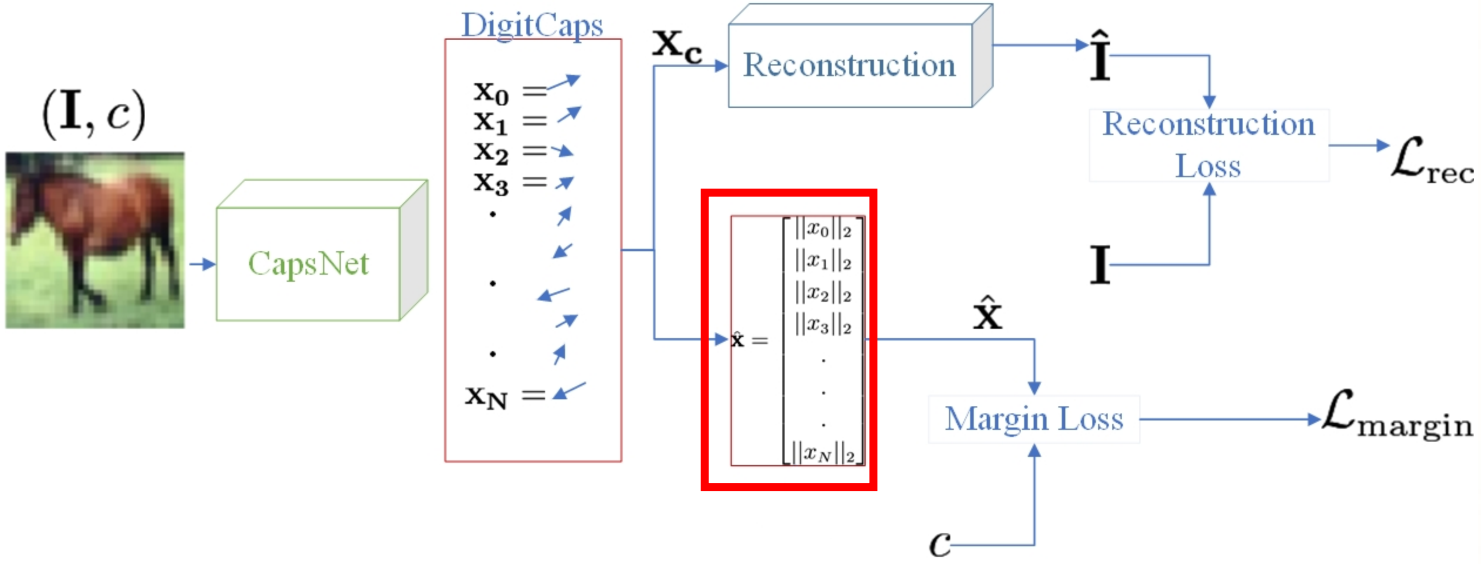
Image embedding and CapsNet

- **An embedding:** is a mapping of a discrete (categorical) variable to a vector of continuous numbers
- **Neural network embeddings:** reduce the dimensionality of categorical variables and meaningfully represent categories in the transformed space
- **CNN-driven approach:** we can obtain images embedding by taking the **intermediate output** (before the classification layer) of a considered architecture

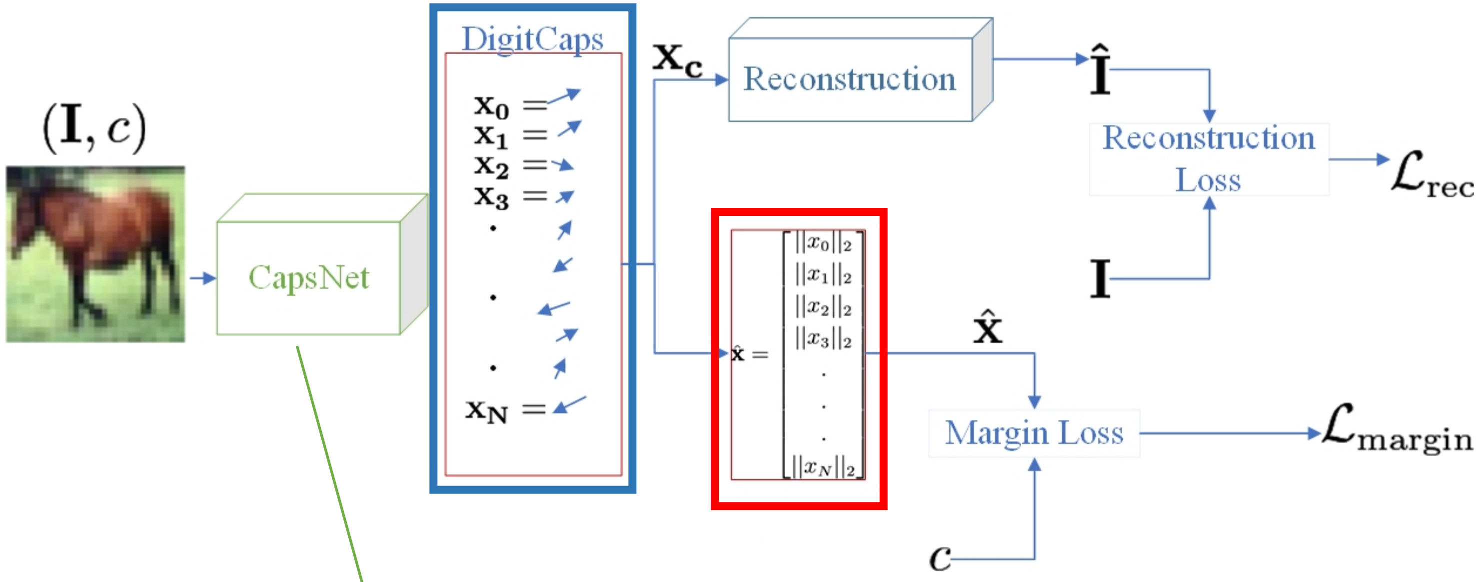


__What if we apply CapsNet as Deep Neural Network encoder to extract the embedding?__



We identify in the magnitude of DigitCaps the **Embedding** of input image

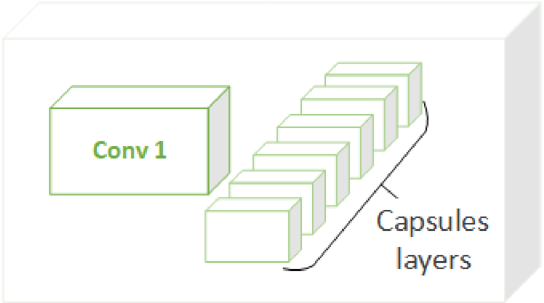
CapsNet overview



DigitCaps is a matrix $X \in R^{NxD}$, N d_dimensional vectors from capsules

N is the number of classes

The image embedding is $\hat{x} = [||x_0||_2 \cdots ||x_N||_2]$: the magnitude of each row (classes) of the matrix X



The structure that characterizes **CapsNet** is a **capsule**, which is a group of neurons generating an activity vector

$\hat{X}$

- **Maintains all the information** from the digits of each capsule
- Provides a **discriminative embedding** based on the predicted class

PolyCapsNet: CapsNet with maximal discriminativity

We want to train our model to enhance the intra-class compactness and inter-class discrepancy among images

Angular margin loss:

reduces the geodesic distance between **the image embedding** ($\hat{x}$) and the **vector centroid** ($\boldsymbol{w_c}$) of the corresponding class c

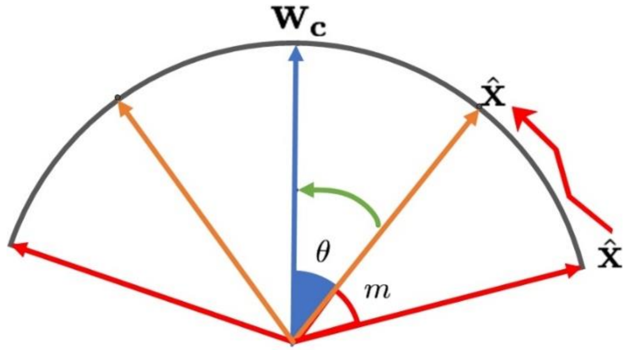
The image embedding ($\hat{x}$)

Softmax Function

$$L_1 = -\frac{1}{K}\sum_{i=1}^{K}\log\frac{e^{\boxed{w_{c_i}^T\hat{x}_i}+b_i}}{\sum_{j=1}^{N}e^{w_j^T\hat{x}_i+b_j}}$$

K: number of input samples

$W \in R^{dxN}$: responsible of projecting the input embedding to the class space



$$w_{c_i}^T\hat{x}_i = \|w_{c_i}\|\|\hat{x}_i\|\cos\theta_J \qquad b = 0 \qquad \text{where } w_c \in W$$

$$L_{angle} = -\frac{1}{K}\sum_{i=1}^{K}\log\frac{e^{s(\cos(\theta_c+m))}}{e^{s(\cos(\theta_c+m))}+\sum_{j=1\,j\neq c}^{N}e^{s(\cos(\theta_j+m))}}$$

<u>Does not encourage the similarity among intra-class samples and the diversity among inter-class samples</u>

Wang, F., et al. "Normface: L2 hypersphere embedding for face verification." (2017).

Polytope stationary embedding

$w_c \in W$

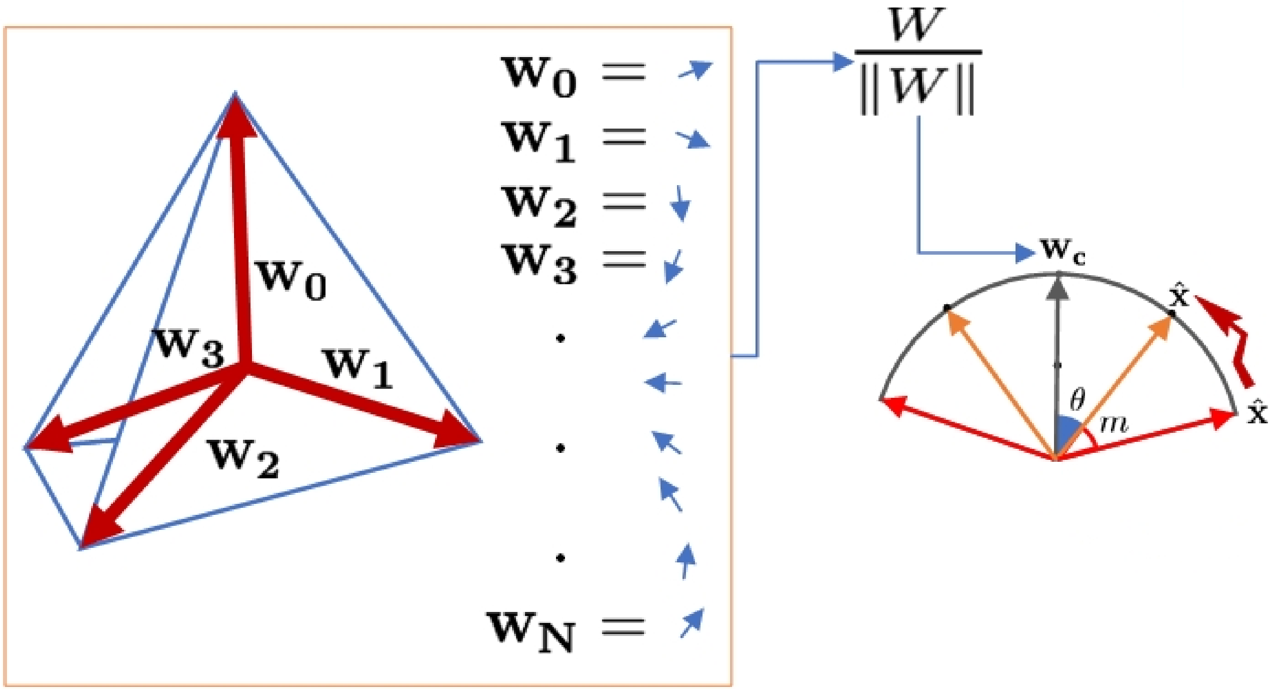
The **angular margin loss** introduces the need for a weight matrix W to represent the **class centroids** described in an hyperspherical space of definition

Fixed Classifier:

- Guaranteeing stationarity of the embedding

Polytopes (geometric object with "flat" sides):

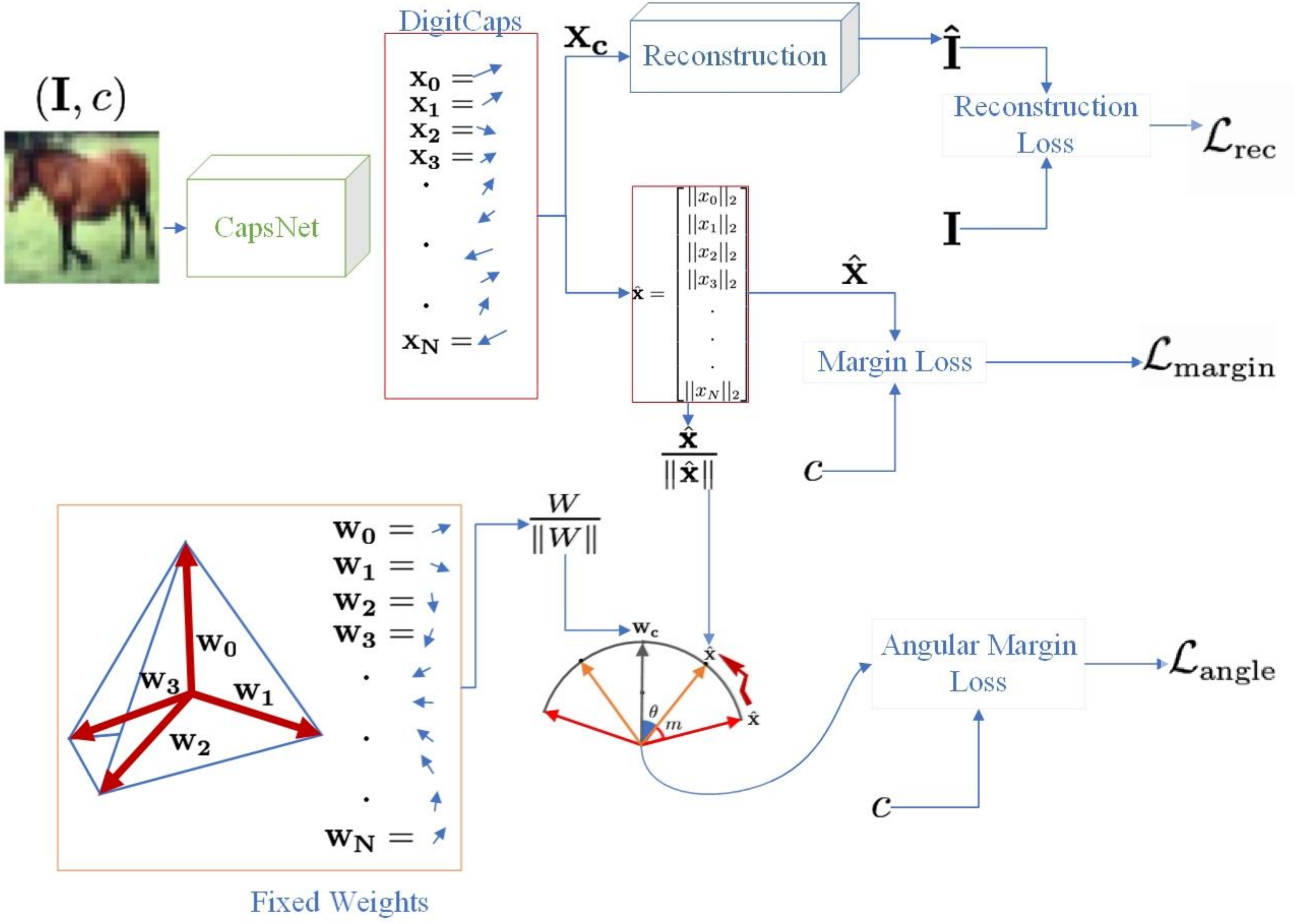
- Define weight matrix
- Maximal discriminability representation



Fixed Weights

Pernici, F., et al. "Fix your features: Stationary and maximally discriminative embeddings using regular polytope (fixed classifier) networks." (2019)

PolyCapsNet



$$\mathcal{L} = \mathcal{L}_{\text{angle}} + \mathcal{L}_{\text{margin}} + \mathcal{L}_{\text{rec}}$$

Classification results

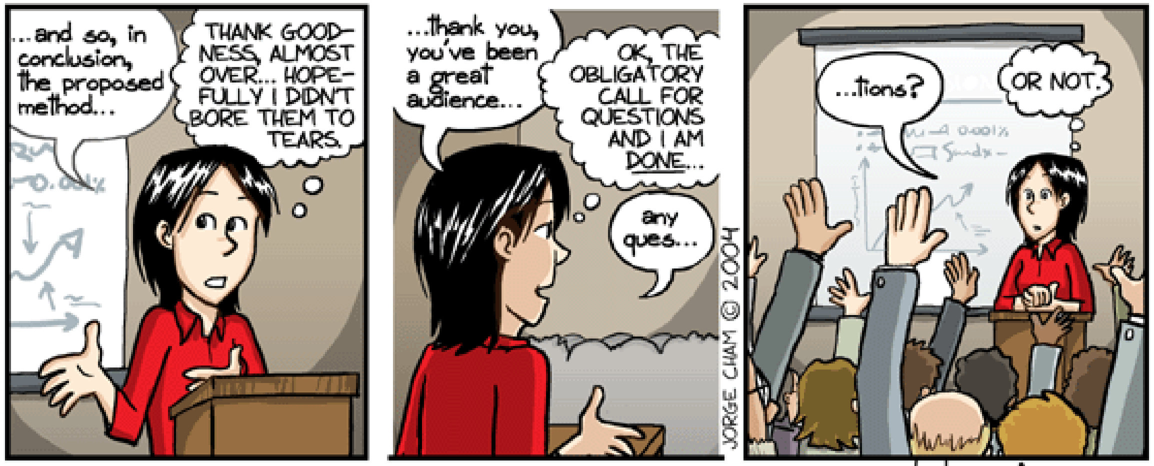
Loss function	MNIST	CIFAR10	SmallNorb
$\mathcal{L}_{\text{margin}}$	99.33%	72.84%	66.98%
$\mathcal{L}_{\text{margin}} + \mathcal{L}_{\text{rec}}$	99.37%	**73.11%**	64.62%
$\mathcal{L}_{\text{angle}}$	99.30%	70.61%	77.29%
$\mathcal{L}_{\text{angle}} + \mathcal{L}_{\text{rec}}$	99.14%	68.97%	66.25%
$\mathcal{L}_{\text{angle}} + \mathcal{L}_{\text{margin}}$	99.39%	68.96%	78.41%
$\mathcal{L}_{\text{angle}} + \mathcal{L}_{\text{margin}} + \mathcal{L}_{\text{rec}}$	**99.39%**	71.61%	**81.44%**

What if we use a learnable W matrix?

Classifier	MNIST	CIFAR10	SmallNorb
Fixed W with Simplex	**99.39%**	**71.61%**	**81.44%**
Learnable W	98.75%	50.38%	57.28%

Questions?

Thank you for listening to this presentation!



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Code: