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# Transferable Model for Shape Optimization subject to Physical Constraints

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## Motivation

**Given:** Fluid dynamical quantities, acting as physical constraints.

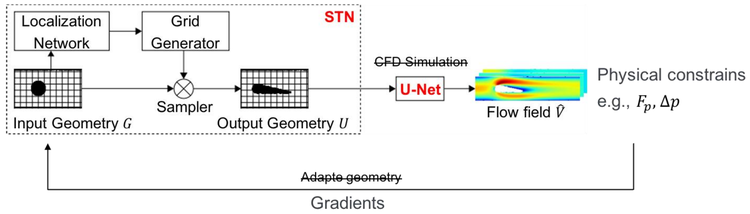
**Target:** Find airfoil geometry, such that constraints are fulfilled.

**Problem:**

- Requires time-consuming CFD-simulations, resulting in a flow field to calculate fluid dynamical quantities e.g., $F_p$, $\Delta p$
- Adapt geometry parameters iterative

**Idea:**

- Replace CFD-simulation with **U-Net** for fast flow field inference
- Spatial Transformer Network (**STN**) to optimize the shape of the airfoil
- Fully differentiable model, so gradients can be used for shape optimization.

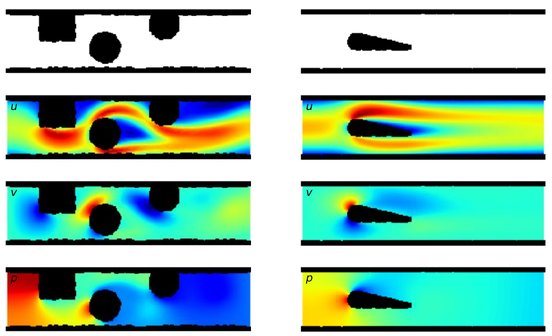



## Dataset

$N = 1350$ simulations of a 2D channel flow with randomly placed geometries inside the channel, consisting of pairs of

- Geometry map $G \in \mathbb{R}^{HxWx1}$
- Flow field $V \in \mathbb{R}^{HxWxC}$

where channels $C$ are holding the values for the velocity in $x$- and $y$-direction $(u, v)$ and the pressure $p$.



The dataset is available at **https://github.com/Flow-Field-Prediction/2D-Channel-Flow**

## Model

Train **U-Net** to infer flow field $\hat{V}$ for a given geometry $U$ by learning the underlying fluid dynamics

$$\mathcal{L}_{SSE} = \sum_{i=1}^{N} \left(\hat{V}_i - V_i\right)^2$$

Train Spatial Transformer Network (**STN**) via physical constraints e.g., pressure forces $F_i$ on the airfoil surface

$$F_i = \int_{A_i} p_i dA_i$$

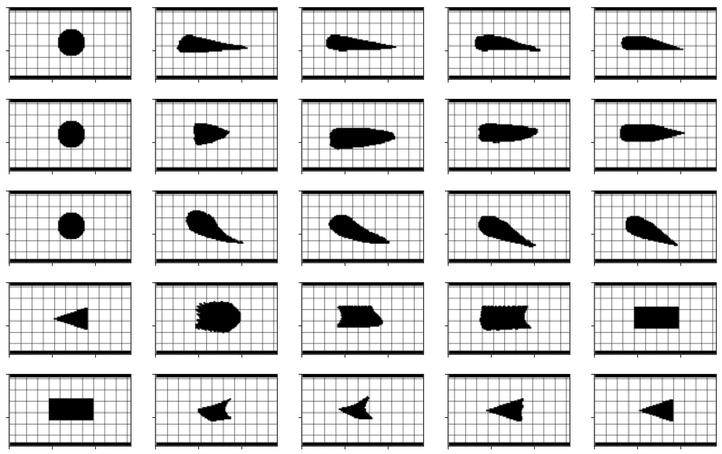
and total pressure difference $\Delta p$

$$\Delta p = \left(\frac{\rho}{2}(|u_2 + v_2|^2) + p_2\right) - \left(\frac{\rho}{2}(|u_1 + v_1|^2) + p_1\right)$$

between channel inlet and outlet.

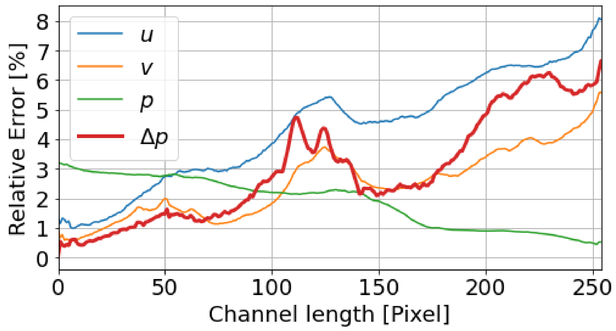
## Geometry Transformation

a) Initial geometry for shape transformation.
b) Constraints $\Delta p, F_i$ lead to broad solution space, where no unique solution can be found.
c) Additional constraining the length (L), height (H) and center of mass (COM) narrows the solution space.
d) Strongest transformation by constraining the whole flow field with the sum of squared errors SSE$(V, \hat{V})$.
e) Target geometry, where the values for the constraints stems from.

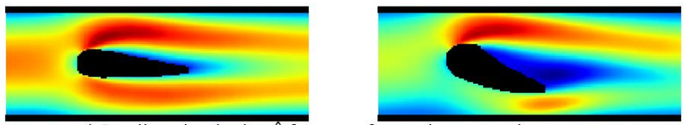


(a) Inital (b) $\Delta p, F_i$ (c) $\Delta p, F_i, L$ $H, COM$ (d) SSE (e) Target

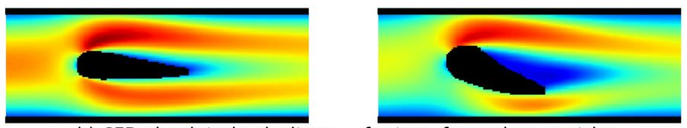
## Flow Field Prediction



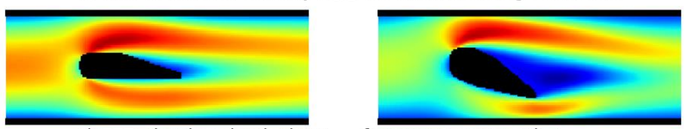

## Comparison with CFD-Simulation



a) Predicted velocity $\hat{u}$ for transformed geometries



b) CFD-simulated velocity $u_{CFD}$ for transformed geometries



c) CFD-simulated velocity $u_{CFD}$ for target geometries