Tensorized Feature Spaces for Feature Explosion

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Introduction

Hyperspectral Images(HSI):

- Images with hundreds of spectral bands at each pixel.
- Used in aerial land surveys with aircrafts or satellites.
- Each pixel has different features corresponding to spectral bands.

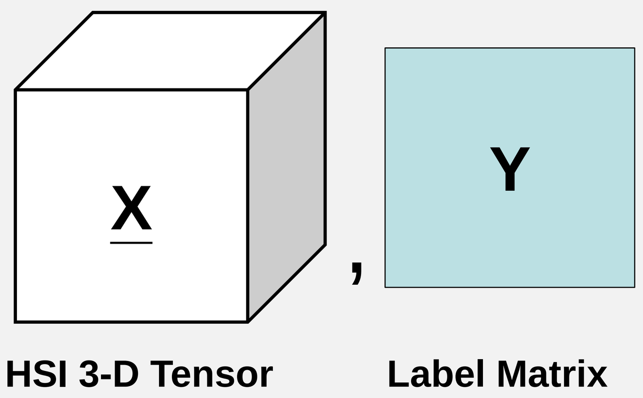
Task: Assuming each pixel belongs to one class, classify all pixels in HSI.

Different spectral bands

Example of Hyperspectral Image[1]

Problem Definition

Given:



, , and Tensor Rank R

Generate a feature space for a classifier such that pixels in the image are classified into one of the given classes.

- **Khatri-Rao Product (KRP)** of two matrices $\mathbf{A} \in \mathbb{R}^{I\times R}$ and $\mathbf{B} \in \mathbb{R}^{J\times R}$ is a column-wise Kronecker product.

$$\mathbf{A} \odot \mathbf{B} = [a_1 \otimes b_1 \; a_2 \otimes b_2 \; \dots a_R \otimes b_R] \in \mathbb{R}^{IJ\times R}$$

- **CP/PARAFAC Decomposition** of a 3-mode tensor of size $I \times J \times K$ for a particular rank R is given by sum of R rank-one tensors:

$$\underline{\mathbf{X}} \approx \sum_{r=1}^{R} \mathbf{A}(:,r) \circ \mathbf{B}(:,r) \circ \mathbf{C}(:,r)$$

Where $\mathbf{A} \in \mathbb{R}^{I\times R}$, $\mathbf{B} \in \mathbb{R}^{J\times R}$ and $\mathbf{C} \in \mathbb{R}^{K\times R}$ are factor matrices.
$\circ$ denotes the three-way outer product.

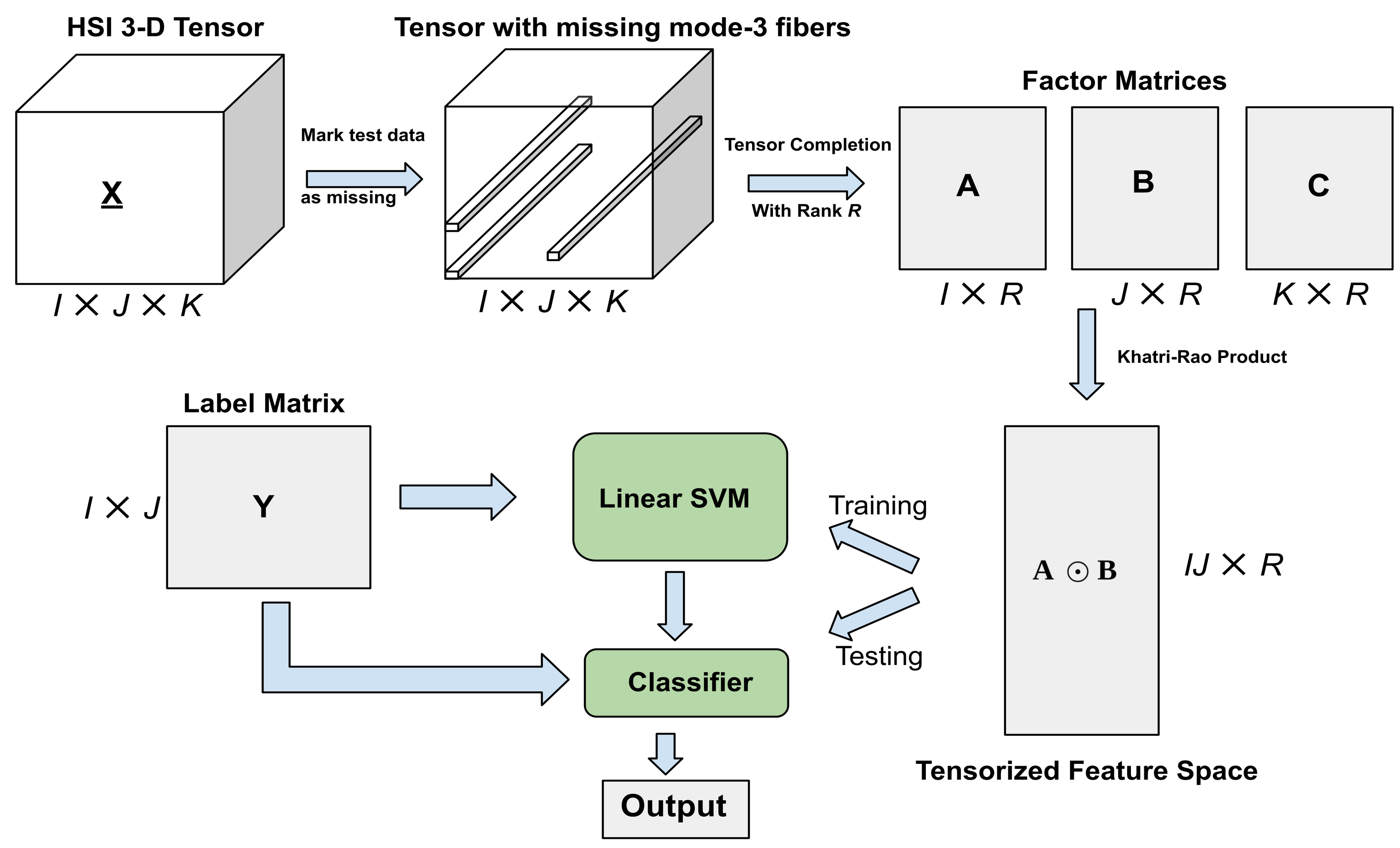
- **Tensor Completion** is the task of predicting missing values in a tensor using tensor decomposition.

Proposed Method: ORION

- **Intuition:** Map the input space to higher dimensional space by exploiting multi-linear structure of tensors.
- CP decomposition of a 3-D tensor $\underline{\mathbf{X}}$ yields 3-factor matrices **A**, **B** and **C.**
- **Tensorized Feature Space:** Khatri-Rao Product of matrices **A** and **B**.

$$\mathbf{A} \odot \mathbf{B} \in \mathbb{R}^{IJ\times R}$$

Overview of ORION



Experimental Evaluation

ORION is implemented using MATLAB and Python. We use the tensor toolbox[4] for tensor completion, scikit-learn[5] for classification tasks and Tensorly[6] for tensor operations in python. All datasets used are publicly available. Code is available at https://github.com/ravdeep003/ORION.

Classification Accuracy

CLASSIFICATION ACCURACY OF ALL THE METHODS FOR 80-20 SPLIT

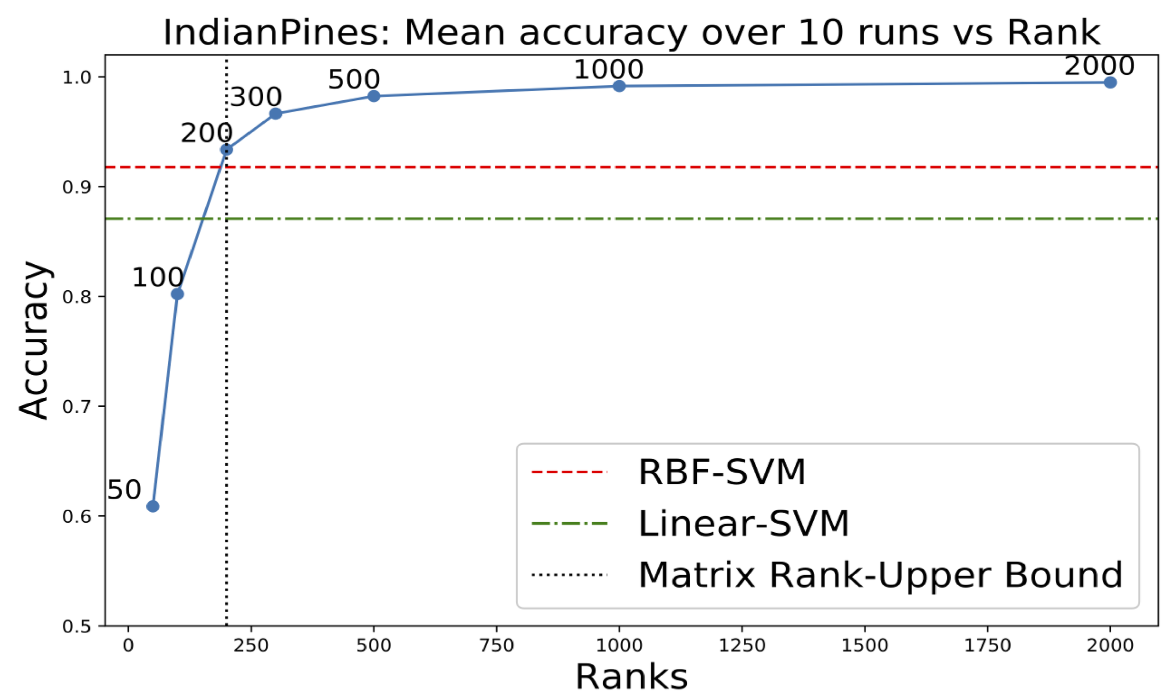
	Indian Pines	Pavia University	Salinas-A	Salinas
Linear SVM	0.8708± 0.0035	0.9176 ± 0.0017	**0.9986 ± 0.0016**	0.9339 ± 0.0014
Polynomial SVM	0.8979± 0.0054	0.9481 ± 0.0015	0.9978 ± 0.0015	0.9463 ± 0.0014
RBF SVM	0.9178± 0.0050	0.9622 ± 0.0020	**0.9985 ± 0.0017**	0.9620 ± 0.0024
MLP	0.9182± 0.0057	**0.9635 ± 0.0041**	0.9982 ± 0.0010	0.9629 ± 0.0045
ORION -1000	**0.9916± 0.0022**	0.9502 ± 0.0032	0.9690 ± 0.0067	**0.9927 ± 0.0010**
ORION -2000	**0.9949± 0.0022**	**0.9828 ± 0.0030**	0.9680 ± 0.0063	**0.9954 ± 0.0006**

- In case of Indian Pines, Pavia University and Salinas datasets, ORION performs better than the baselines.

CLASSIFICATION ACCURACY OF ALL THE METHODS FOR 30-70 SPLIT

	Indian Pines	Pavia University	Salinas-A	Salinas
Linear SVM	0.8371 ± 0.0034	0.9134 ± 0.0015	**0.9965 ± 0.0010**	0.9322 ± 0.0007
Polynomial SVM	0.8511 ± 0.0042	0.9367 ± 0.0010	0.9941 ± 0.0017	0.9406 ± 0.0009
RBF SVM	0.8739 ± 0.0041	**0.9546 ± 0.0007**	**0.9966 ± 0.0011**	0.9515 ± 0.0012
MLP	0.8693 ± 0.0098	**0.9556 ± 0.0029**	0.9931 ± 0.0029	0.9475 ± 0.0041
ORION -1000	**0.9725 ± 0.0032**	0.9119 ± 0.0015	0.8607 ± 0.0146	**0.9662 ± 0.0013**
ORION -2000	**0.9806 ± 0.0031**	0.9544 ± 0.0021	0.8982 ± 0.0073	**0.9832 ± 0.0013**

- One of the challenges in HSI pixel classification is limited labelled data. We split the data into 30% training and 70% testing.
- In case of Indian Pines and Salinas ORION performs better than baselines.
- In both cases, baselines perform better in Salinas-A. A probable explanation for this behavior is Salinas-A, which is a subscene of Salinas, has a linearly separable structure. We shall investigate this in future work.



- As the Tensor rank increases, accuracy increases until a certain point. As we conjecture this increase in rank results in explosion of feature space which resembles how kernel method works.

Conclusions

- Introduced tensorized feature space based on factors generated from tensor decomposition.
- Demonstrated effectiveness of our methods against traditional linear and non-linear supervised learning methods.

References

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[1] Image from: https://en.wikipedia.org/wiki/Hyperspectral_imaging