# Matching of Matching-Graphs - A Novel Approach for Graph Classification 

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## Introduction

Graphs are recognized as versatile alternative to feature vectors. That is, graphs are used in diverse applications (protein function/structure prediction, signature verification or detection of Alzheimer's Disease). A large amount of graph based methods for pattern recognition have been proposed. Graph edit distance (GED) is one of the most flexible distance models available. GED generally offers more information than merely a dissimilarity score, namely the information of the objects actually match with each other (known as edit path).

## Matching-Graphs

Given two graphs $g_{1}$ and $g_{2}$, the basic idea of GED is to transform $g_{1}$ into $g_{2}$ using some edit operations (e.g. insertions, deletions, and substitutions of both nodes and edges). The set of operations used to transform $g_{1}$ into $g_{2}$ is called edit path. For each edit path, two matching-graphs $m_{g_{i} \times g_{j}}$ and $m_{g_{j} \times g_{i}}$ can eventually be built (for the source and the target graph $g_{i}$ and $g_{j}$, respectively). To this end, all nodes that are substituted are added to the matching-graphs. All nodes that are deleted or inserted are neither considered in the two matching-graphs. Nodes without adjacent nodes are removed from the resulting matching-graphs.
In Fig. 1 we can see an example of the source matching-graph created from the example edit path $\lambda=\{0 \rightarrow 0,1 \rightarrow 1,2 \rightarrow$ $2,3 \rightarrow 3,4 \rightarrow 4,5 \rightarrow \varepsilon, \varepsilon \rightarrow 5, \varepsilon \rightarrow 6, \varepsilon \rightarrow 7\}$.


Figure 1: Visualization of an edit path and the resulting matching-graphs.

To handle the edges, we propose two different approaches: pruned and unpruned matching-graphs. For a pruned matching-graph if two nodes $u_{1}, u_{2} \in V_{i}$ of a source graph $g_{i}$ are substituted with nodes $v_{1}, v_{2} \in V_{j}$ in a target graph $g_{j}$ and there is an edge $\left(u_{1}, u_{2}\right) \in E_{i}$ available, $\left(u_{1}, u_{2}\right)$ is actually included in the matching-graph $m_{g_{i} \times g_{j}}$ if, and only if, there is an edge $\left(v_{1}, v_{2}\right)$ available in $E_{j}$. In an unpruned matching-graph it is included regardless whether or not edge $\left(v_{1}, v_{2}\right)$ is available in $E_{j}$.

## Classification using matching-graphs

The novel matching-graphs are employed in a distance based classification scenario. We define a distance measure that combines two distances with each other. A Visual illustration of this process can be seen in Fig. 2. The orange part $d_{B P}\left(g, g_{i}\right)$ is the distance information between a given test graph $g$ and a training graph $g_{i}$, calculated using the BP approximation algorithm for GED. The green part $\min _{m \in \hat{\mathcal{A}}_{\omega_{l}}} d_{B P}(g, m)$ is the minimum of all distances from $g$ to all matching graphs $m$ that are of the same class as $g_{i}$. We use $\alpha \in[0,1]$ as a weighting parameter.


Figure 2: Example of the distance calculation using matching-graphs

## Results

The reference system, denoted as $d_{B P}$, is based on the GED approximation algorithm BP in conjunction with the training graphs only. Our system, denoted as $d_{M}$, uses the same GED approximation yet makes use of the novel matching-graphs.

|  | $k-\mathrm{NN}\left(d_{B P}\right)$ | $k-\mathrm{NN}\left(d_{M}\right)$ |  |
| :--- | :--- | :--- | :--- |
| Data Set |  | Unpruned | Pruned |
| Letter | 90.5 | 91.3 | $93.1 \circ$ |
| AIDS | 99.0 | $99.7 \circ$ | $99.7 \circ$ |
| Mutagenicity | 70.6 | 70.0 | 70.5 |

Table 1: Results Table. On the left the baseline and on the right our approach. The o indicates a statistically significant improvement over the reference system using a Z-Test at significance level $\alpha=0.05$

We observe that the distance-based classification achieves better results with $d_{M}$ rather than $d_{B P}$ on two data sets (Letter and AIDS) and with both strategies (pruned and unpruned edges). On the Mutagenicity data set a slight non statistically significant - deterioration is observed with our novel approach.
Hence, we conclude that the integration of matching-graphs can lead to a more accurate determination of a dissimilarity score using GED.

