## Motivations

In robotics, registration of 3D point sets is a key issue in localization applications. The trend for autonomous vehicles makes it a widely search field.
In this application the 3D point clouds are captured with a LiDAR and the registration is based solely on this information.
Iterative Closest Point (ICP) [1] is one of the mostly used algorithms for 3D point clouds registration, but, the point matching step is known to be time consuming due to the potential large number of points.

## Objective

The aim of the presented algorithm is to find the transformation ${ }^{t} \mathbf{T}_{s}$ that best fits a source point cloud to a target point cloud. The transformation is defined as follows:

$$
{ }^{t} \mathbf{T}_{s}=\left[\begin{array}{cc}
{ }^{t} \mathbf{R}_{s} & { }^{t} \mathbf{t}_{s} \\
\mathbf{0}_{3 \times 1} & 1
\end{array}\right]
$$

with ${ }^{t} \mathbf{R}_{s}$ and ${ }^{t} \mathbf{t}_{s}$ respectively a $3 \times 3$ rotation matrix and a $3 \times 1$ translation vector.
Man-made environment, such as buildings, are usually composed of strong planar structures. Planes are less numerous than points and give a good representation of the captured scene. Thus, similarly to the classical ICP algorithm, the proposed algorithm iteratively performs a matching step and a minimization step, but instead of points, planes are registered.


Fig. 1 Example of a registration between two point clouds (scans from ASL dataset [2]). In white the target point cloud - In green the source point cloud. Left. before registration - Right: after registration.

## Distances Definitions

To find ${ }^{t} \mathbf{T}_{s}$, the plane-to-plane distance is minimized. To ensure an accurate registration, an additional point-to-plane registration is added at the end of the process (Fig.4). Both minimization are performed using a nonlinear Gauss-Newton optimization

Plane-to-plane distance:

$$
\mathbf{d}_{i}^{\Pi}=\binom{{ }^{t} \mathbf{R}_{s}^{s} \mathbf{n}_{i}-{ }^{t} \mathbf{n}_{i}}{\left.{ }_{[t} \mathbf{R}_{s}{ }^{s} \mathbf{n}_{i}\right]^{\top}{ }^{\top} \mathbf{t}_{s}+{ }^{s} \rho_{i}-{ }^{t} \rho_{i}}
$$

where ${ }^{s} \mathbf{n}_{i}$ and ${ }^{t} \mathbf{n}_{i}$ are the normal to the planes ${ }^{s} \Pi_{i}$ and ${ }^{t} \Pi_{i}$, respectively, and ${ }^{s} \rho_{i}$ and ${ }^{t} \rho_{i}$ their respective distance to the origin of the sensor in the target frame.

Point-to-plane distance:

$$
d_{i}^{\perp}=\left\|{ }^{t} \mathbf{n}_{i}^{\top} \cdot\left({ }^{t} \mathbf{T}_{s}{ }^{s} \mathbf{p}_{i}-{ }^{t} \mathbf{p}_{i}\right)\right\|^{2}
$$

with $^{s} \mathbf{p}_{i}$ and ${ }^{t} \mathbf{p}_{i}$, respectively the source and target point, and ${ }^{t} \mathbf{n}_{i}$ the surface normal computed from ${ }^{{ }^{2}}{ }_{i}$ neighborhood


Fig. 2 Plane segmentation example. Left: input point cloud - Right: plane extraction result. Each extracted plane is in a different color. Red points are outliers.

## Plane Matching

For each extracted plane ${ }^{s} \Pi_{i}$ in the source, a list of planes in the target, that are potential matches for the source plane, is made. Each target candidate ${ }^{t} \Pi_{j}$ is given a score within the range $[0,1]$. It is computed from the following features:

- the distance between the projections of the origin of the planes:

$$
d_{o}=\left\|^{s} \rho_{i}{ }^{s} \mathbf{n}_{i}-{ }^{t} \rho_{j}{ }^{t} \mathbf{n}_{j}\right\|^{2}
$$

- the distance between the centroids of the planes:

$$
d_{c}=\left\|^{s} \overline{\mathbf{p}}_{i}-{ }^{t} \overline{\mathbf{p}}_{j}\right\|^{2}
$$

with ${ }^{s} \overline{\mathbf{p}}_{i}$ and ${ }^{t} \overline{\mathbf{p}}_{j}$ the centroids of source end target planes.

- the area ratio between the planes:

$$
S_{r}=\frac{\min \left({ }^{s} S_{i},{ }^{t} S_{j}\right)}{\max \left({ }^{s} S_{i},{ }^{t} S_{j}\right)}
$$

- the dot product of the normals of the planes:

$$
\phi_{n}={ }^{s} \mathbf{n}_{i} \cdot{ }^{t} \mathbf{n}_{j}
$$

Each feature is normalized between $[0,1]$ (denoted further) and weighted, leading to a score defined as follows:
score $=\alpha \cdot \hat{d}_{o}+\beta \cdot \hat{d}_{c}+\gamma \cdot\left(1-\hat{S}_{r}\right)+\delta \cdot\left(1-\hat{\phi}_{n}\right)$
with:
$\alpha+\beta+\gamma+\delta=1$

A target plane is considered as a valid match if it respects the following condition: score $<t_{\text {score }}$

## Results <br> Comparison with state-of-the-art algorithms:

 Using the indoor sequences of the Autonomous System Labs (ASL) [2] dataset, the distance of the estimated pose to the ground truth is evaluated with the Euclidean distance $\Delta_{\mathrm{t}}$ and the geodesic distance $\Delta_{r}$ (Fig.3):$$
\begin{gathered}
\Delta_{t}=\left\|^{t} \hat{\mathbf{t}}_{s}-{ }^{t} \mathbf{t}_{s}^{*}\right\| \\
\Delta_{r}=\arccos \left(\frac{\operatorname{trace}\left({ }^{t} \mathbf{R}_{s}^{*-1}{ }^{*} \hat{\mathbf{R}}_{s}\right)-1}{2}\right)
\end{gathered}
$$

With ${ }^{t} \mathbf{R}_{s}^{*}$ and ${ }^{t} \mathbf{t}_{s}^{*}$ respectively the ground truth rotation matrix and translation vector. The thresholds to estimate a successful registration are 0.1 m for translation and $2.5^{\circ}$ for rotation, as suggested in [4].


Tab. 1 Percentage of successful registration (translation and rotation combined) for the evaluated algorithms on each considered sequence [2].

| Sequence | Proposed <br> method | G-ICP[5] | NDT[4] | ICP-PCL |
| :--- | :--- | :--- | :--- | :--- |
| Apartment | $\mathbf{1 0 0}$ | 75 | 77 | 43 |
| ETH | $\mathbf{1 0 0}$ | $\mathbf{1 0 0}$ | $\mathbf{1 0 0}$ | $\mathbf{1 0 0}$ |
| Stairs | $\mathbf{1 0 0}$ | 97 | 97 | 90 |

Impact of the two-step minimization:


Fig. 3 Cumulative probabilities of : (top) translation error (in meters) and (bottom) rotation error (in degrees) for Apartment sequence on each evaluated algorithm. The vertical bars represent the thresholds (respectively 0.1 m for translation and $2.5^{\circ}$ for rotation) for successful registration.

Fig. 4 3D mapping of the Apartment sequence using the proposed method. White: the ground truth trajectory. Purple dots: plane-to-plane only registration trajectory. Red dots. combination of plane-to-plane and point-to-plane registration

