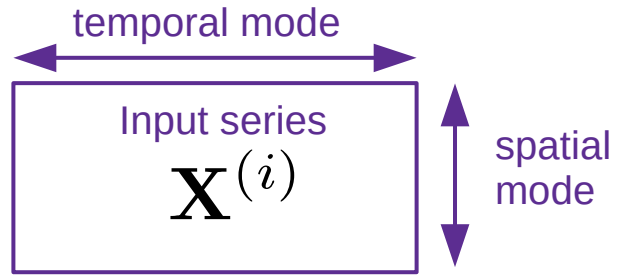


# Data Normalization for Bilinear Structures in High-Frequency Financial Time-series



spatial mode normalization

$$\mathbf{x}_1^{(i)}(1) \quad \text{-----}$$

$$\mathbf{x}_1^{(i)}(d) \quad \text{-----}$$

$$\mathbf{x}_1^{(i)}(D) \quad \text{-----}$$

$$\bar{\mathbf{x}}_1^{(i)} = \text{mean}(\mathbf{x}_1^{(i)}(d)), \forall d = 1, \dots, D$$

$$\sigma_1^{(i)} = \text{std}(\mathbf{x}_1^{(i)}(d)), \forall d = 1, \dots, D$$

$$\mathbf{Z}_1^{(i)} = (\mathbf{X}^{(i)} - \mathbf{1}_D(\bar{\mathbf{x}}_1^{(i)})^T)(\mathbf{1}_D(\sigma_1^{(i)})^T)$$

$$\tilde{\mathbf{X}}_1^{(i)} = (\mathbf{1}_D\gamma_1^T) \odot \mathbf{Z}_1^{(i)} + \mathbf{1}_D\beta_1^T$$

temporal mode normalization

$$\mathbf{x}_2^{(i)}(1) \quad \text{---} \quad \mathbf{x}_2^{(i)}(t) \quad \text{---} \quad \mathbf{x}_2^{(i)}(T)$$

$$\bar{\mathbf{x}}_2^{(i)} = \text{mean}(\mathbf{x}_2^{(i)}(t)), \forall t = 1, \dots, T$$

$$\sigma_2^{(i)} = \text{std}(\mathbf{x}_2^{(i)}(t)), \forall t = 1, \dots, T$$

$$\mathbf{Z}_2^{(i)} = (\mathbf{X}^{(i)} - \bar{\mathbf{x}}_2^{(i)}\mathbf{1}_T^T)(\sigma_2^{(i)}\mathbf{1}_T^T)$$

$$\tilde{\mathbf{X}}_2^{(i)} = (\gamma_2\mathbf{1}_T^T) \odot \mathbf{Z}_2^{(i)} + \beta_2\mathbf{1}_T^T$$

normalized series

$$\tilde{\mathbf{X}}^{(i)} = \lambda_1 \tilde{\mathbf{X}}_1^{(i)} + \lambda_2 \tilde{\mathbf{X}}_2^{(i)}$$

Learnable Parameters via SGD:  $\gamma_1, \beta_1, \gamma_2, \beta_2, \lambda_1, \lambda_2$