A Heuristic-Based Decision Tree for Connected Components Labeling of 3D Volumes

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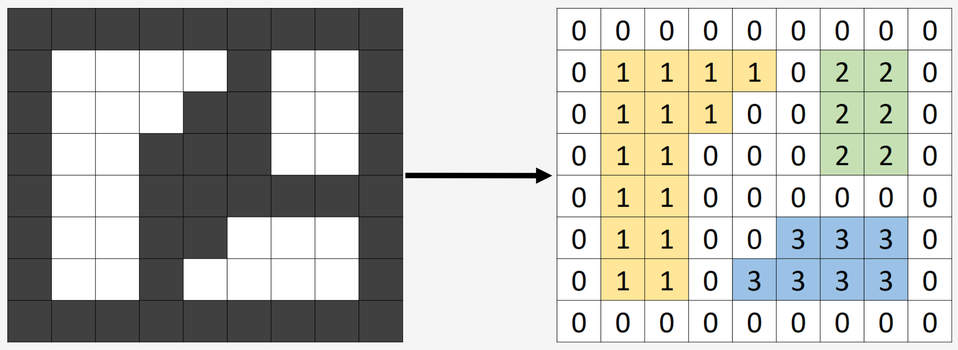
Connected Components Labeling (CCL)

- Find all connected, foreground pixel regions within a binary image
- Each pixel region, or **connected component**, receives a unique label
- Fundamental for image segmentation and object recognition
- CCL should be as <u>fast</u> as possible



0	0	0	0	0	0	0	0	0
0	1	1	1	1	0	2	2	0
0	1	1	1	0	0	2	2	0
0	1	1	0	0	0	2	2	0
0	1	1	0	0	0	0	0	0
0	1	1	0	0	3	3	3	0
0	1	1	0	3	3	3	3	0
0	0	0	0	0	0	0	0	0

History of CCL Research

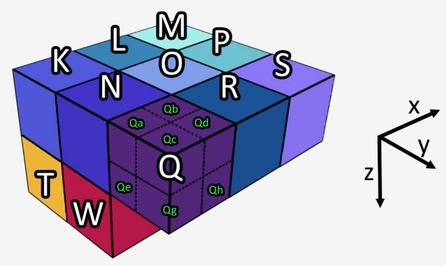
- Rosenfeld and Pfaltz invented **two scans** algorithms
- Wu et al. proposed **Optimal Decision Trees (ODTs)**
- Grana et al. proposed **block-based mask**
- What about 3D CCL?
  - Multiple possible block-based masks: 2x1x1, 2x2x1 and 2x2x2
  - Explosion in complexity makes the ODT generation **infeasible**
  - Existing 3D CCL algorithms **do not employ** block-based masks
  - **Goal: generate a near-optimal tree with a heuristic strategy**

p	**q**	**r**
s	**x**	

Rosenfeld mask

(a)	**b**	**c**	**d**	**e**	(f)
g	**h**	**i**	**j**	**k**	(l)
m	**n**	**o**	**p**		
(q)	**r**	**s**	**t**		

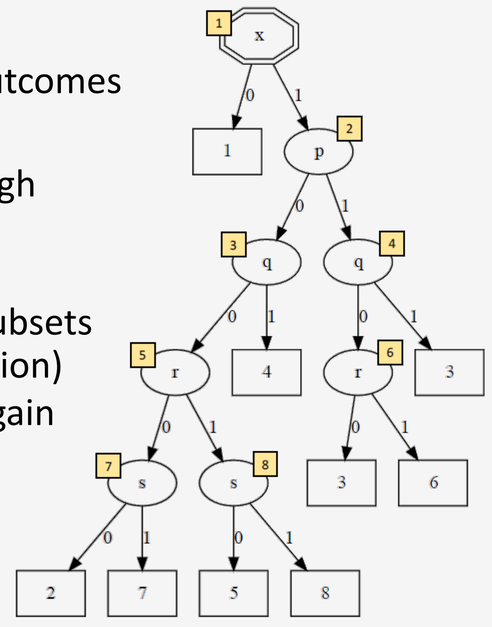
2x2 Grana mask



2x2x2 voxel mask

Heuristics – Concept

- **Shannon Entropy** (information theory)
  - Given a set of events E, with P_i being the probability of an event $i \in E$, the entropy H_E is:
    $$H_E = \sum_i -P_i \log P_i$$
  - Entropy describes the uncertainty of outcomes
- **Decision Tree Learning**
  - **Recursively partition** the dataset through entropy calculation
    1. Try *splitting* on every attribute
    2. Calculate **Information Gain (IG)** on subsets (IG measures average entropy reduction)
    3. Apply *split* with highest information gain
- **Entropy Partitioning Decision Tree (EPDT)** for the Rosenfeld mask is **near-optimal**



Node	Depth	$H(S)$	p			q			r			s			x		
			H_0	H_1	IG	H_0	H_1	IG	H_0	H_1	IG	H_0	H_1	IG	H_0	H_1	IG
1	0	2.2	2.0	1.4	0.5	2.3	1.5	0.3	1.9	2.1	0.2	2.1	2.1	0.1	**0.0**	**2.4**	**1.0**
2	1	2.4	**2.0**	**0.8**	**1.0**	2.5	1.0	0.7	1.8	2.3	0.4	2.2	2.2	0.2			
3	2	2.0				**2.0**	**0.0**	**1.0**	1.5	1.5	0.5	1.5	1.5	0.5			
4	2	0.8				**1.0**	**0.0**	**0.3**	0.0	1.0	0.3	0.8	0.8	0.0			
5	3	2.0							**1.0**	**1.0**	**1.0**	1.0	1.0	1.0			
6	3	1.0							**0.0**	**0.0**	**1.0**	1.0	1.0	0.0			
7	4	1.0										**0.0**	**0.0**	**1.0**			
8	4	1.0										**0.0**	**0.0**	**1.0**			

Applying Decision Tree Learning to 3D CCL

- New 3D EPDT CCL algorithms
- Varying block size and number of pixels

- **EPDT_19c**
  - Block size 2x1x1
  - Smallest 3D block-based mask

Ka	**Kb**	**La**	**Lb**	**Ma**	Mb
Na	**Nb**	**Oa**	**Ob**	**Pa**	Pb
Qa	**Qb**	**Ra**	**Rb**	**Sa**	Sb

Ta	**Tb**	**Ua**	**Ub**	**Va**	Vb
Wa	**Wb**	**Xa**	**Xb**		

- **EPDT_22c**
  - Block size 2x1x1
  - Add borders pixels, for more efficient actions

Ka	**Kb**	**La**	**Lb**	**Ma**	Mb
Na	**Nb**	**Oa**	**Ob**	**Pa**	**Pb**
Qa	**Qb**	**Ra**	**Rb**	**Sa**	Sb

Ta	**Tb**	**Ua**	**Ub**	**Va**	Vb
Wa	**Wb**	**Xa**	**Xb**		

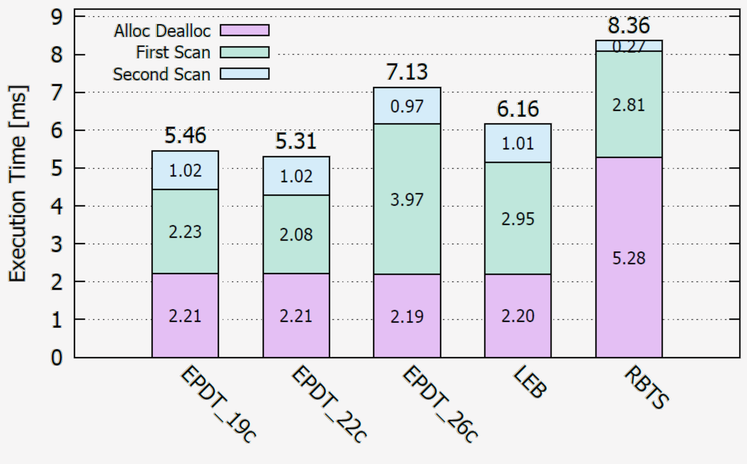
- **EPDT_26c**
  - Block size 2x2x1
  - Largest tree that compilers can handle

Ka	Kb	La	Lb	Ma	Mb
Kc	**kd**	**Lc**	**Ld**	**Mc**	Md
Na	**Nb**	**Oa**	**Ob**	**Pa**	Pb
Nc	**Nd**	**Oc**	**Od**	**Pc**	Pd
Qa	**Qb**	**Ra**	**Rb**	**Pa**	Pb
Qc	Qd	Rc	Rd	Pc	Pd

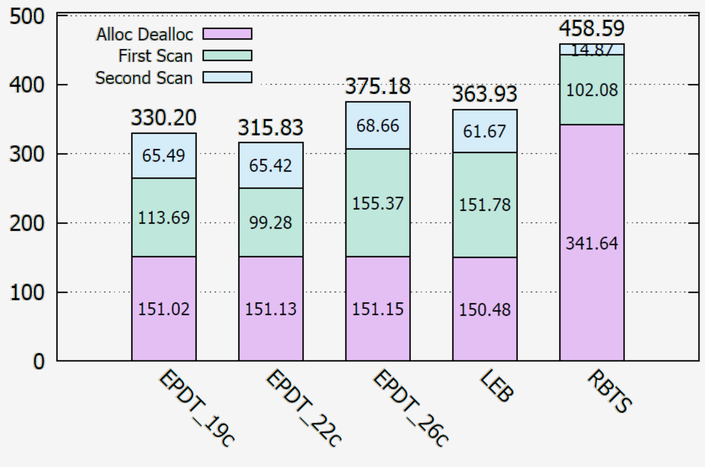
Ta	Tb	Ua	Ub	Va	Vb
Tc	**Td**	**Uc**	**Ud**	**Vc**	Vd
Wa	**Wb**	**Xa**	**Xb**		
Wc	**Wd**	**Xc**	**Xd**		

Experimental Results

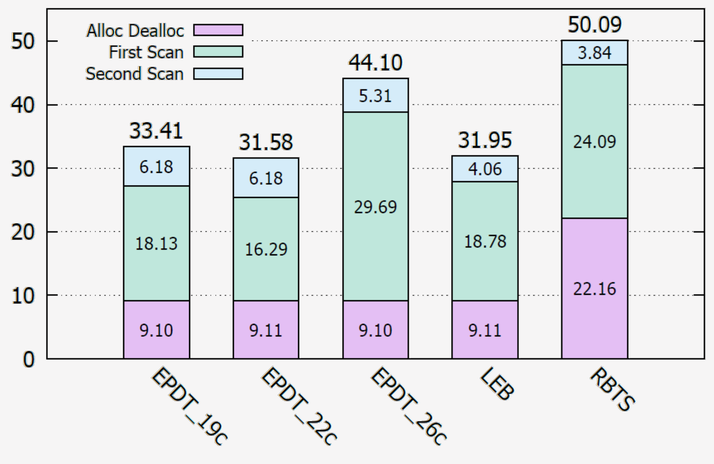
Hilbert



Mitochondria



OASIS



- EPDT algorithms improve the performance of the first scan by saving many **memory accesses**
- EPDT_26c has a very large decision tree → **bad impact** on instruction cache
- EPDT_22c **improves** current state-of-the-art[1]

Average number of load/store operations on the OASIS dataset, expressed in millions.

Algorithm	Binary Image	Labels Image	Equivalences Vector	Total
LEB	11.461	27.182	9.851	48.494
EPDT_19c	14.917	17.760	1.169	33.846
EPDT_22c	14.057	17.753	1.145	32.955
EPDT_26c	13.695	13.145	0.728	27.568

[1]L. He, Y. Chao, and K. Suzuki, "Two Efficient Label-Equivalence-Based Connected-Component Labeling Algorithms for 3-D Binary Images," *IEEE Transactions on Image Processing*, vol. 20, no. 8, pp. 2122–2134, 2011.