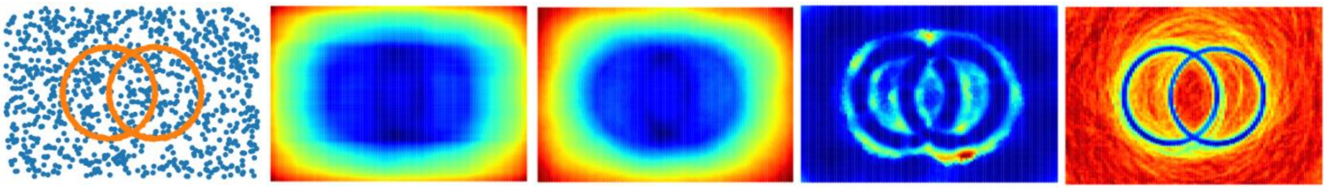
PIF: Anomaly detection via preference embedding

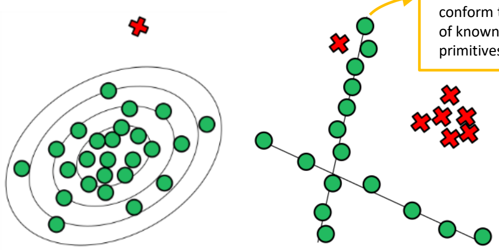
Filippo Leveni* filippo.leveni@polimi.it
Luca Magri* luca.magri@polimi.it
Giacomo Boracchi* giacomo.boracchi@polimi.it
Cesare Alippi*† cesare.alippi@polimi.it cesare.alippi@usi.ch

* POLITECNICO MILANO 1863
†



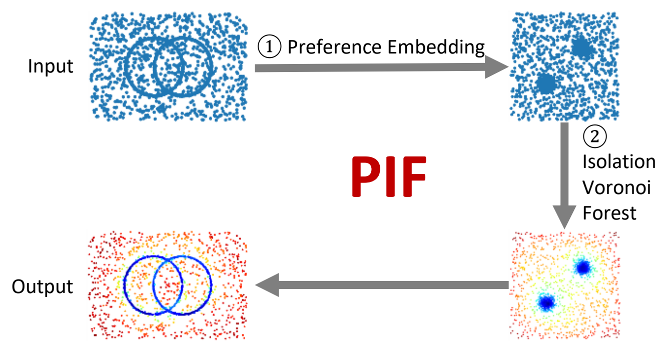
Ground truth | IFOR [1] | EIFOR [2] | LOF [3] | PIF

Goal: Detect anomalies as those samples that **deviate from unknown structures or patterns**.



Structures/patterns conform to a family of known geometric primitives.

Our solution: Preference Isolation Forest

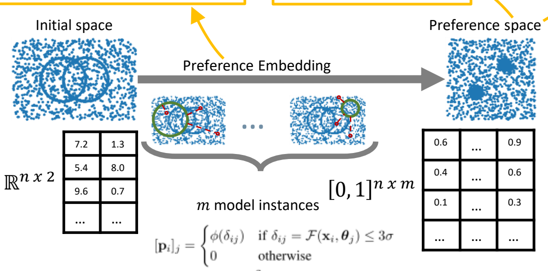


① ***Preference Embedding***: m model instances are sampled and preferences of each point are collected. This yields an embedding from $\mathbb{R}^{n \times 2}$ to $[0,1]^{n \times m}$.

② ***Isolation Voronoi Forest***: k nested Voronoi tessellations are built in the preference space using Tanimoto distance τ. A function α_ψ of the average path length is used as anomaly score.

Preference embedding is used to identify what conform to structures and what are the anomalies.

In the preference space structures can be easily identified.



$$[\mathbf{p}_i]_j = \begin{cases} \phi(\delta_{ij}) & \text{if } \delta_{ij} = \mathcal{F}(\mathbf{x}_i, \boldsymbol{\theta}_j) \leq 3\sigma \\ 0 & \text{otherwise} \end{cases}$$

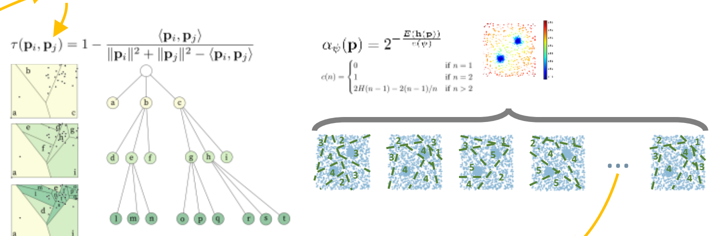
$$\phi(\delta) = \exp(-\delta^2/\sigma)$$

Preference space is equipped with the Tanimoto distance.

$$\tau(\mathbf{p}_i, \mathbf{p}_j) = 1 - \frac{\langle \mathbf{p}_i, \mathbf{p}_j \rangle}{\|\mathbf{p}_i\|^2 + \|\mathbf{p}_j\|^2 - \langle \mathbf{p}_i, \mathbf{p}_j \rangle}$$

$$\alpha_\psi(\mathbf{p}) = 2^{-\frac{E(h(\mathbf{p}))}{c(\psi)}}$$

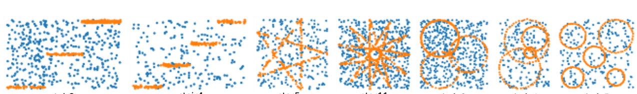
$$c(n) = \begin{cases} 0 & \text{if } n = 1 \\ 1 & \text{if } n = 2 \\ 2H(n-1) - 2(n-1)/n & \text{if } n > 2 \end{cases}$$



An Isolation Voronoi Tree employs the isolation principle in the preference space.

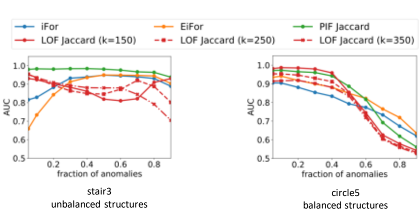
Isolation Voronoi Forest is very efficient.

Experiments: PIF **outperforms all the alternatives**, both on synthetic and real [4] data, where anomaly-detection methods are straightforwardly plugged in the preference space.



stair3 | stair4 | star5 | star11 | circle3 | circle4 | circle5

	Euclidean				Preference binary				Preference			
	LOF ℓ_2	iFor	EiFor	PIF ℓ_2	LOF jac	iFor	EiFor	PIF jac	LOF tani	iFor	EiFor	PIF
stair3	0.737	0.925	0.920	0.918	0.904	0.885	0.864	0.958	0.815	0.923	0.925	**0.971**
stair4	0.814	0.889	0.874	0.871	0.849	0.855	0.860	0.941	0.881	0.912	0.908	**0.952**
star5	0.771	0.722	0.738	0.788	0.875	0.745	0.769	0.872	**0.929**	0.761	0.822	0.910
star11	0.671	0.728	0.727	0.738	0.830	0.739	0.741	0.771	**0.900**	0.738	0.774	0.796
circle3	0.761	0.698	0.732	0.779	0.719	0.842	0.854	0.900	0.731	0.854	0.891	**0.930**
circle4	0.640	0.641	0.665	0.679	0.827	0.686	0.699	0.860	0.906	0.667	0.720	0.897
circle5	0.543	0.569	0.570	0.633	0.699	0.597	0.617	0.672	**0.823**	0.573	0.593	0.780
Mean	0.705	0.739	0.747	0.772	0.815	0.764	0.772	0.853	0.855	0.775	0.805	**0.891**



stair3
unbalanced structures

circle5
balanced structures

Preference embedding increases the separability between structured and unstructured data.

johnsona

	LOF tani	iFor	EiFor	PIF
barrsmith	**0.969**	0.708	0.715	0.944
bonhall	0.918	0.969	0.967	0.949
bonython	**0.978**	0.679	0.691	0.954
elderhalla	0.999	0.925	0.909	**0.999**
elderhallb	0.986	0.924	0.943	**0.999**
hartley	0.963	0.749	0.793	**0.989**
johnsona	0.993	0.993	0.993	**0.998**
johnsonb	0.776	0.999	0.998	0.999
ladysymon	0.847	0.944	0.943	**0.997**
library	**1.000**	0.764	0.771	0.998
napiera	0.975	0.869	0.879	**0.983**
napierb	0.888	0.931	0.936	**0.953**
neem	0.985	0.896	0.906	**0.996**
nese	**0.996**	0.888	0.892	0.980
oldclassicswing	0.936	0.923	0.943	**0.987**
physics	0.670	0.858	0.787	**1.000**
sene	**0.997**	0.698	0.731	0.988
unihouse	0.785	0.998	0.998	**0.999**
unionhouse	**0.987**	0.639	0.664	0.968
Mean	0.929	0.861	0.866	**0.983**

Homographies

biscuitbookbox

	LOF tani	iFor	EiFor	PIF
biscuit	0.976	0.994	0.996	**1.000**
biscuitbook	1.000	0.987	0.988	1.000
biscuitbookbox	**1.000**	0.990	0.989	0.996
boardgame	**0.962**	0.400	0.304	0.949
book	0.996	1.000	1.000	1.000
breadcartoychips	**0.989**	0.978	0.971	0.976
breadcube	**1.000**	0.998	0.998	0.999
breadcubechips	**0.999**	0.985	0.985	0.998
breadtoy	0.984	0.999	0.998	0.999
breadtoycar	**0.998**	0.933	0.883	0.991
carchipscube	**0.993**	0.981	0.966	0.987
cube	0.999	0.970	0.982	0.999
cubebreadtoychips	0.990	0.962	0.958	0.989
cubechips	**1.000**	0.995	0.994	1.000
cubetoy	**1.000**	0.997	0.995	1.000
dinobooks	0.887	0.873	0.857	**0.899**
game	**1.000**	0.901	0.895	0.999
gamebiscuit	1.000	0.985	0.988	**1.000**
toycubecar	**0.973**	0.290	0.192	0.964
Mean	0.987	0.906	0.891	0.987

Fundamental matrices

[1] F. T. Liu, K. M. Ting, and Z.-H. Zhou, "Isolation forest", in International Conference on Data Mining, IEEE, 2008, pp. 413–422.
[2] S. Hariri, M. C. Kind, and R. J. Brunner, "Extended isolation forest", arXiv preprint arXiv:1811.02141, 2018.
[3] M. M. Breunig, H.-P. Kriegel, R. T. Ng, and J. Sander, "Lof: Identifying density-based local outliers", in International Conference on Management of data, 2000, pp. 93–104.
[4] H. S. Wong, T.-J. Chin, J. Yu, and D. Suter, "Dynamic and hierarchical multi-structure geometric model fitting," in International Conference on Computer Vision, 2011.