A Randomized Algorithm for Sparse Recovery

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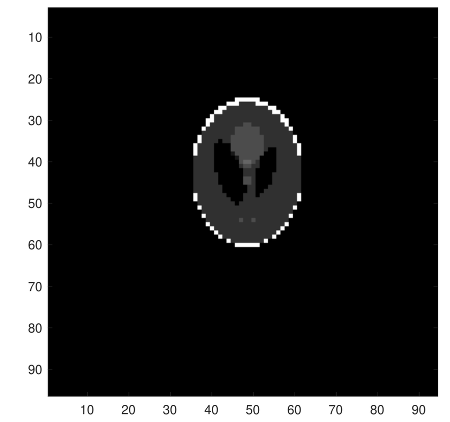
Sparse Signal Recovery

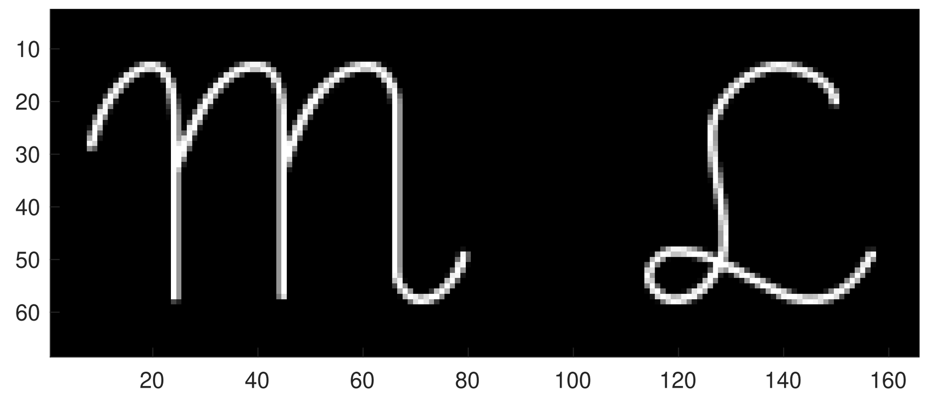
- Finding the solution to an underdetermined linear system: $\hat{x} = arg \min_{x \in R^n} \|Ax - y\|$.
- There are k non-zeros in x, and $k << n$
- Exploiting the sparsity of a signal to recover it from far fewer samples than required by the Nyquist-Shannon sampling theorem.

Examples

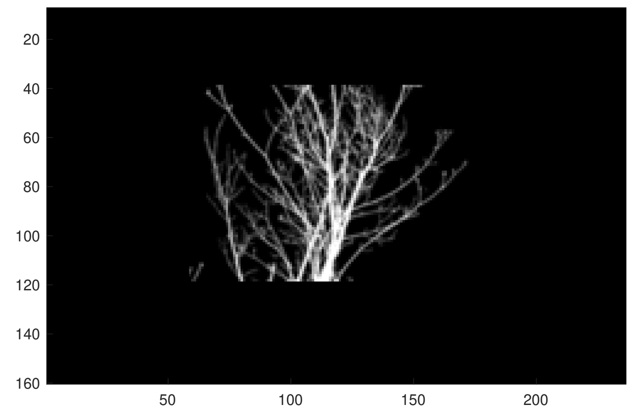
Sparsity: $641/9024$



Sparsity: $851/11424$



Sparsity: $4890/38400$



Algorithm

Outline of the Proposed Algorithm

$$
\begin{array}{l}
\text{Initialization: } x^0 \leftarrow 0 \\
\textbf{for } i = 0, \dots, T \textbf{ do} \\
\quad b^i \leftarrow A^T(y - Ax^i) \\
\quad S \leftarrow \text{supp}(\mathcal{T}(b^i)) \\
\quad \Gamma \leftarrow S \cup \text{supp}(x^i) \\
\quad z|_\Gamma \leftarrow A_\Gamma^\dagger y,\ z|_{\Gamma^C} \leftarrow 0 \\
\quad x^{i+1} \leftarrow z \\
\textbf{end for} \\
\text{Return } x^{i+1}
\end{array}
$$

Approach

- Leveraging the structure in the signal can recover a sparse signal with fewer number of measurements
- A k-sparse signal can be recovered using only $O(k)$ measurements
- Use a constrained EMD model
- Find the solution to a minimum-cost flow problem
- Use a randomized algorithm with $\mathbb{E}(c_H) = \mathbb{E}(c_T) = 1$
- Reduce algorithm complexity by using only one invocation of the model projection operator $\mathcal{T}$ for both head and tail approximations

Results

- A randomized algorithm with geometric convergence
- Mean of the output is optimal
- Relaxed the isometric requirement for sparse signal recovery:
  - A has RIP, or
  - A is scalable to a matrix that satisfies RIP