Generalized Conics: Properties and Applications

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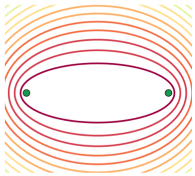
Generalized Conics level sets that extend properties of the conic sections, such as parabola, hyperbola and ellipse, by accepting infinitely many focal points

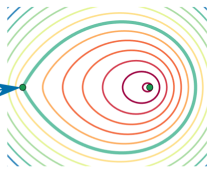
Ellipse

- constant sum of the weighted distances to **two** focal points



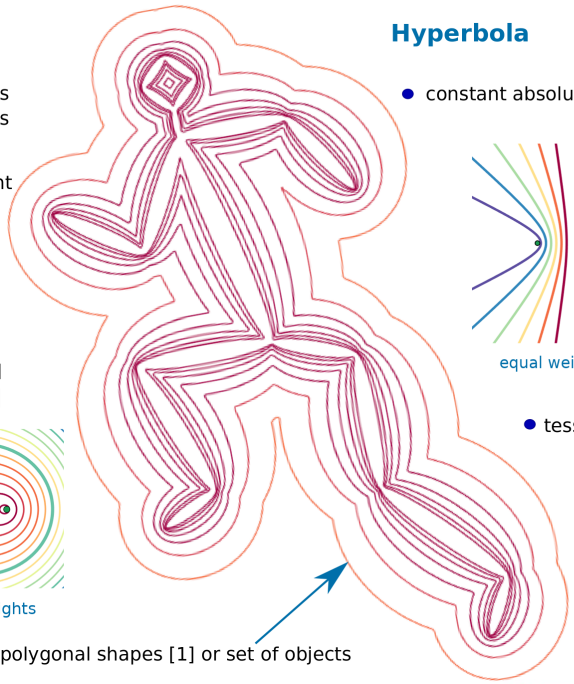
equal weights

- distance from point to line segment - Confocal Ellipse-based Distance (CED) [1]
- invariant to rotation, translation, and discretization of the line segment [1]
- generates ovals and egg-shapes with corners



non-equal weights

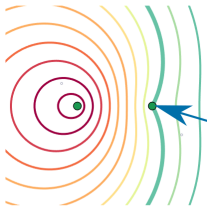
- efficient distance field computation for polygonal shapes [1] or set of objects



Hyperbola

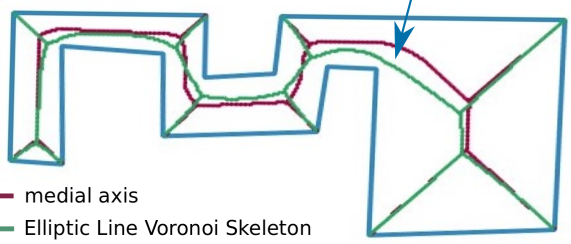
- constant absolute difference of the weighted distances to **two** focal points

equal weights



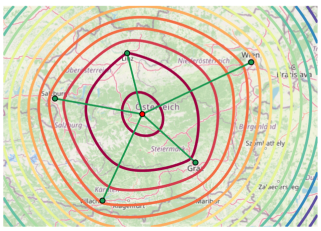
non-equal weights

- generates concavities
- tessellates the space and represents the shape: Elliptic Line Voronoi Diagram & Skeleton [2]
- enables shape smoothing that affects mainly high frequences and can preserve sharp corners [2]
- optimizes the route w.r.t. edge length

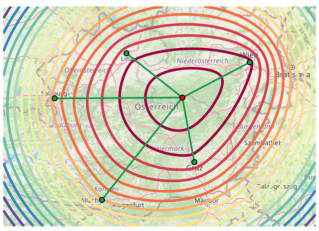


Multifocal Ellipse

- constant sum of the weighted distances to **N** focal points
- level sets are convex : single global minimum except for the even number of collinear focal points
- solves Fermat-Torricelli and Weber problem [3] in linear time



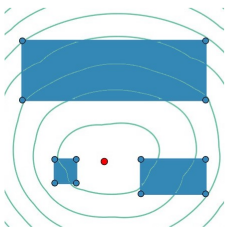
equal weights



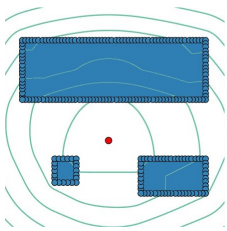
non-equal weights

Extended view on focal points

- can be a point, line segment, or shape : requires normalization of the corresponding distance field
- generalized conic can be a focal point : promotes hierarchical representation



CED

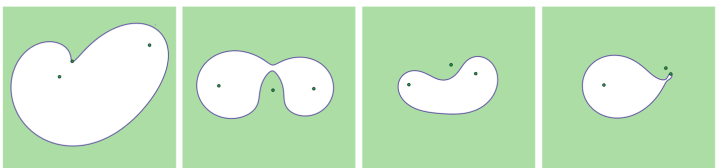


Euclidean distance

Multifocal ellipses, where the foci are rectangles. Global minimum is at the red point.

Multifocal Hyperbola

- constant absolute difference of two multifocal ellipses
- tessellates the space according to the total weighted distances of two sets of focal points
- level sets are not necessarily convex
- generates shapes with concavities



various shapes generated from three focal points with positive and negative weights

References

[1] A. Gabdulkhakova and W. G. Kropatsch, “Confocal ellipse-based distance and confocal elliptical field for polygonal shapes,” in 24th International Conference on Pattern Recognition (ICPR), 2018, pp. 3025–3030.

[2] A. Gabdulkhakova, M. Langer, B. W. Langer, and W. G. Kropatsch, "Line Voronoi Diagrams using elliptical distances", Joint IAPR International Workshops on Statistical Techniques in Pattern Recognition and Structural and Syntactic Pattern Recognition. 2018. pp. 258-267.

[3] C. Groß, T. K. Strempel, "On generalizations of conics and on a generalization of the Fermat-Torricelli problem", The American mathematical monthly, 105(8), 732-743.

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