# A Globally Optimal Method for the PnP Problem with MRP Rotation Parameterization

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# Introduction

- We are concerned with efficiently estimating the pose (i.e., rotation + translation) of a calibrated pinhole camera given the image projections of n ≥ 3 known 3D points
- This is known as the *perspective-n-point (PnP)* problem and is a long-studied one in vision
- PnP arises often in computer vision, photogrammetry and robotics applications involving localization, pose tracking and multi-view 3D reconstruction
- This work develops a direct least squares, algebraic solution that combines a global optimality condition with the modified Rodrigues parameters (MRPs) representation for rotation
- This leads to a system of eighth degree polynomials that is solved using the Gröbner basis approach

# Our Contribution

- Nakano has developed an optimality condition independent of any rotation parameterization; illustrated it with Cayley
- We combine Nakano's optimality condition with the modified Rodrigues parameters to parameterize rotation
- MRPs: G. Terzakis, M. Lourakis, and D. Ait-Boudaoud, "Modified Rodrigues parameters: An efficient representation of orientation in 3D vision and graphics", JMIV 2018
- Our solution:
  - Is the first to apply MRPs to develop a minimal solver for a geometric vision problem
  - Doubles the domain of validity of the Cayley representation by using MRPs with no impact on accuracy and only a moderate impact on computational performance
  - Provides a principled way of dealing with singularities in the representation of rotation

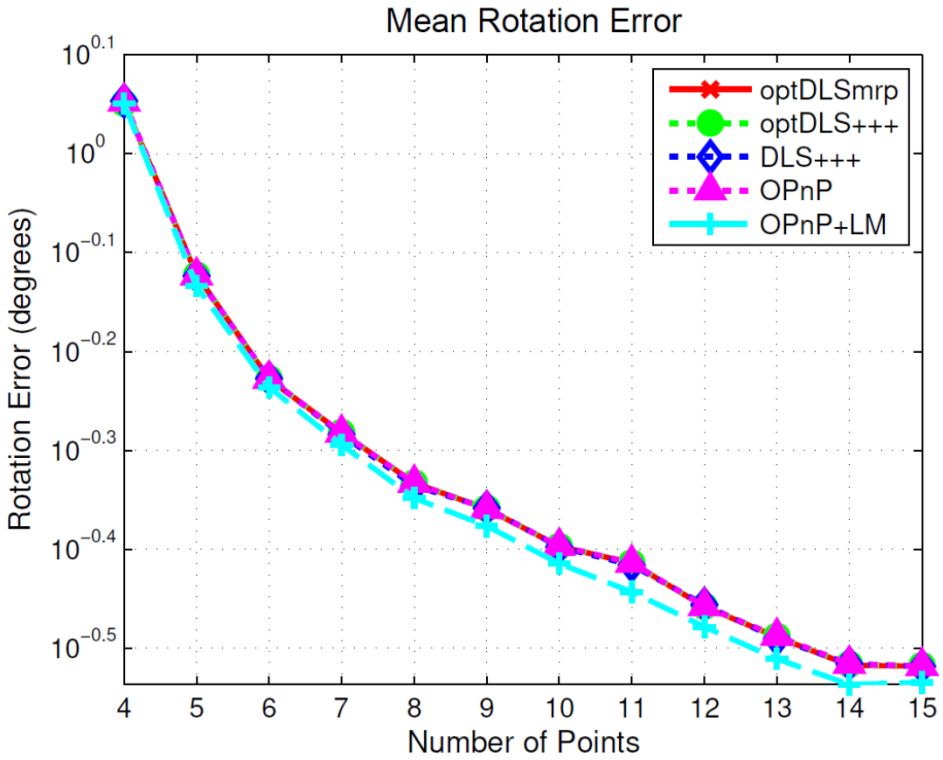
# Experimental Evaluation

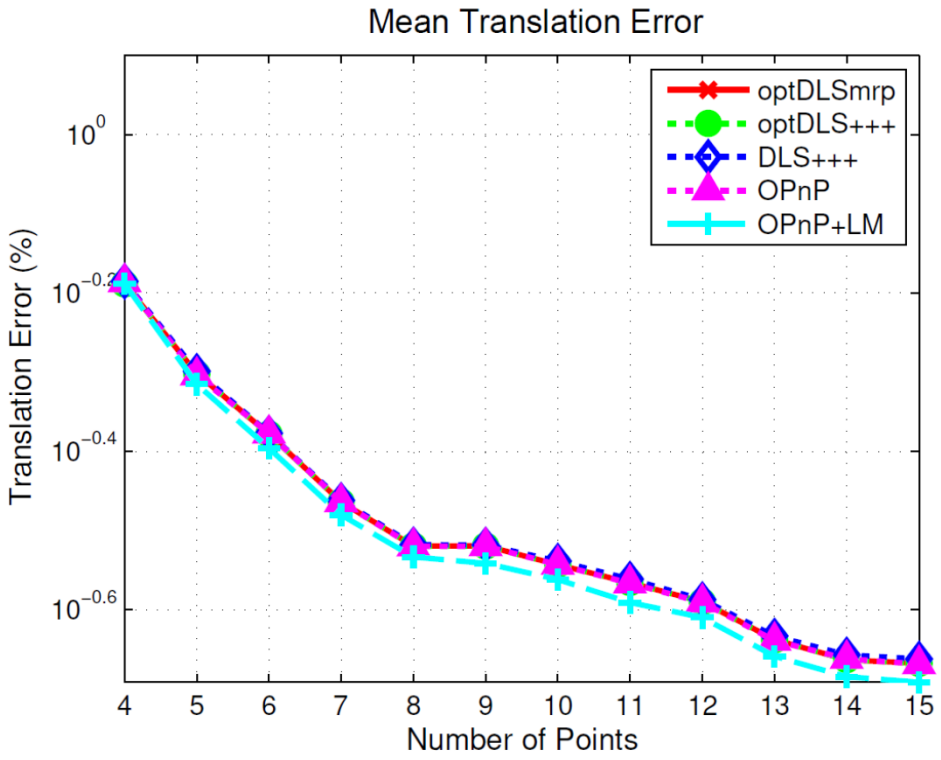
- We used simulated data to compare our MRP-based method (denoted optDLSmrp) against:
  - Nakano's optDLS [1]
  - Hesch and Roumeliotis' DLS [2]
  - Zheng et al.'s OPnP [3]
  - OPnP followed by non-linear minimization of the reprojection error with Levenberg-Marquardt non-linear least squares (OPnP+LM)
- Details of the testing framework are in the paper
- 500 independent trials are executed for each set of parameters


1. *G. Nakano: "Globally optimal DLS method for PnP problem with Cayley parameterization", BMVC 2015*
2. *J. A. Hesch and S. I. Roumeliotis, "A direct least-squares (DLS) method for PnP", ICCV 2011*
3. *Y. Zheng, Y. Kuang, S. Sugimoto, K. Åström, and M. Okutomi, "Revisiting the PnP problem: A fast, general and optimal solution", ICCV 2013*

# Experimental Evaluation (cont'd)

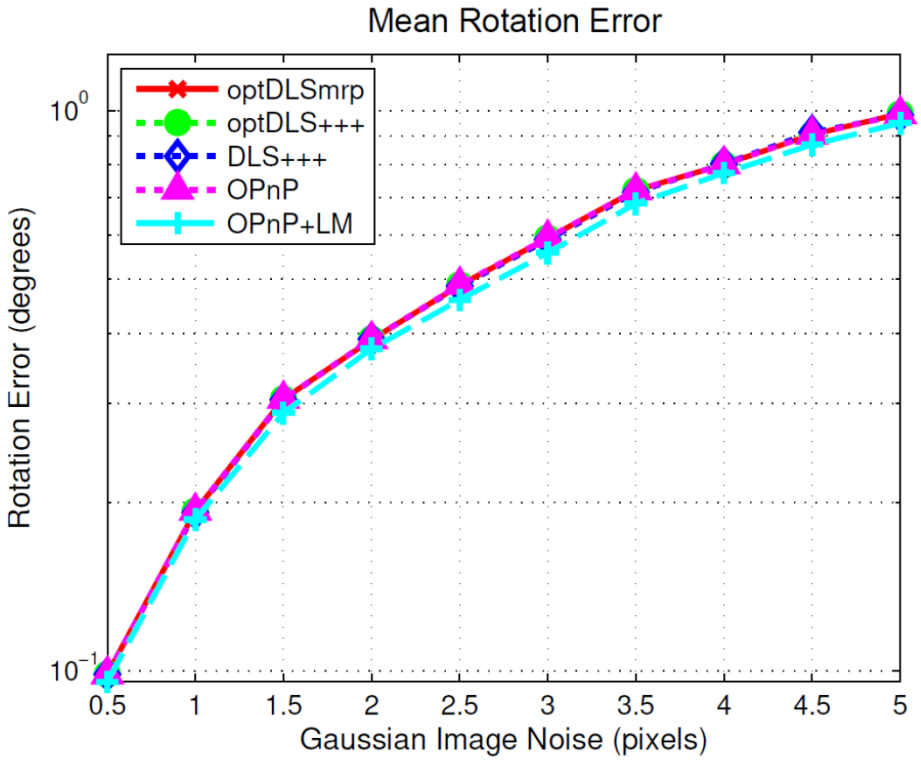
- Mean rotation and translation error for n random points, 4≤n≤15 and additive zero-mean Gaussian noise of standard deviation σ=2 pixels; y-axes are logarithmic

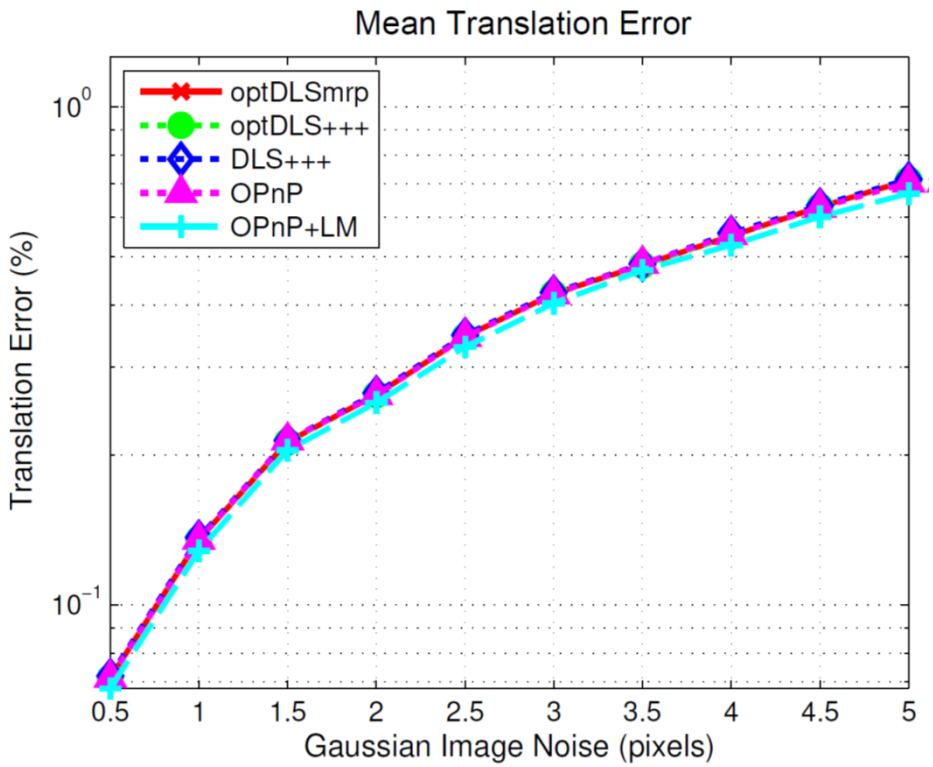


# Experimental Evaluation (cont'd)

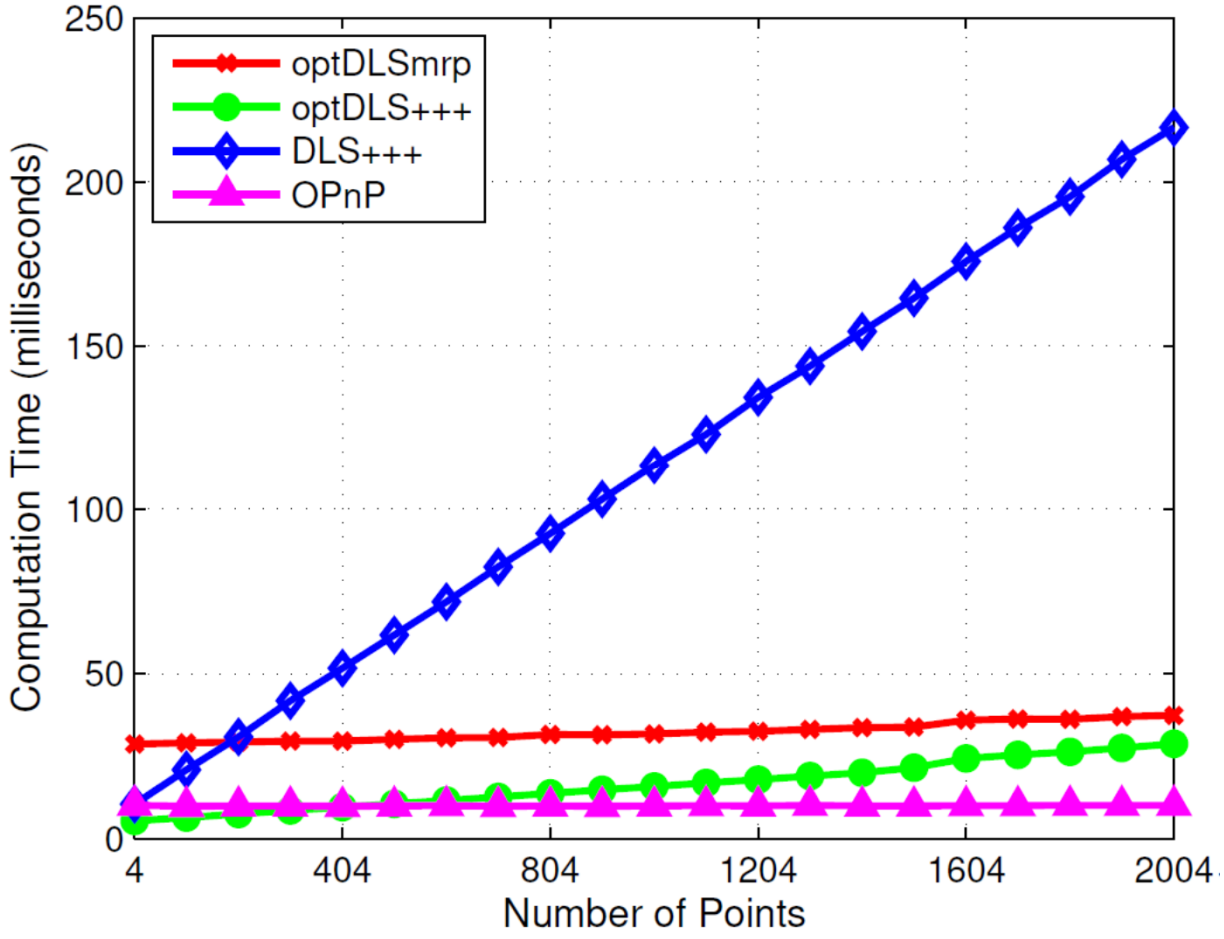
- Mean rotation and translation error for n=10 random points, contaminated with additive zero-mean Gaussian noise with standard deviation $0.5 \leq \sigma \leq 5$ pixels



# Experimental Evaluation (cont'd)

- Average execution times (in ms, under MATLAB) of the various solvers for n ∈ {4, 104, 204, ..., 2004} random points and additive zero-mean Gaussian noise of standard deviation σ=2 pixels



# Summary & Conclusions

- Presented a globally optimal direct least squares method for the PnP problem
- The method couples Nakano's optimality condition with the MRPs representation of rotation to derive a system of polynomial equations
- A novel solution to PnP is obtained by solving this system with Gröbner basis methods and combining it with sequential rotations to avoid the full rotation singularity of MRPs
- Experiments with simulated data have demonstrated that our solution attains accuracy that is indistinguishable from Nakano's Cayley-based method with twice the rotational range and only a moderate increase in computational cost
- MATLAB code is available