Probabilistic Latent Factor Model for Collaborative Filtering with Bayesian Inference

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Problem(1)

- Latent Factor Model (LFM) is one of the most successful methods for Collaborative Filtering (CF) in the recommendation system, in which both users and items are projected into a joint latent factor space.

- LFM models user-item interactions as inner products of factor vectors of user and item in that space and can be efficiently solved by least square methods with optimal estimation.

- However, such optimal estimation methods are prone to overfitting due to the extreme sparsity of user-item interactions.

Problem(2)

- The recommendation problem is formulated as a problem of predicting unobserved rating.

$$\hat{r}_{ui} = f(r_{ui}|\mathbf{p}_u, \mathbf{q}_i) = \mathbf{p}_u^T \mathbf{q}_i = \sum_{k=1}^{K} p_{uk} q_{ik}$$

overfitting

- The optimal estimation method learns the model parameters of LFM by minimizing the regularized squared error.

$$\min_{\mathbf{p}_*, \mathbf{q}_*} \sum_{(u,i,r_{ui}) \in \mathbf{D}} (r_{ui} - \mathbf{p}_u^T \mathbf{q}_i)^2 + \lambda(\|\mathbf{p}_u\|^2 + \|\mathbf{q}_i\|^2)$$

Methodology(1)

- Prior Assumption

$$p(p_{uk}|\mu_{uk},\sigma_{uk}) \sim \mathcal{N}(\mu_{uk},\sigma_{uk}),$$

$$p(q_{ik}|\mu_{ik},\sigma_{ik}) \sim \mathcal{N}(\mu_{ik},\sigma_{ik}).$$

$$p(b_u|\overline{r}) \sim \mathcal{N}(0,\overline{r}),$$

$$p(b_i|\overline{r}) \sim \mathcal{N}(0,\overline{r}).$$

$$b_{ui} = \overline{r} + b_u + b_i.$$

- The likelihood function of BLFM

$$p(r_{ui}|\mathbf{p}_u,\mathbf{q}_i) \sim \mathcal{N}(\sum_{k=1}^{K} p_{uk}q_{ik},\overline{r}).$$

- The likelihood function of BLFMBias

$$p(r_{ui}|\mathbf{p}_u,\mathbf{q}_i,b_u,b_i) \sim \mathcal{N}(\sum_{k=1}^{K} p_{uk}q_{ik} + b_{ui},\overline{r}).$$

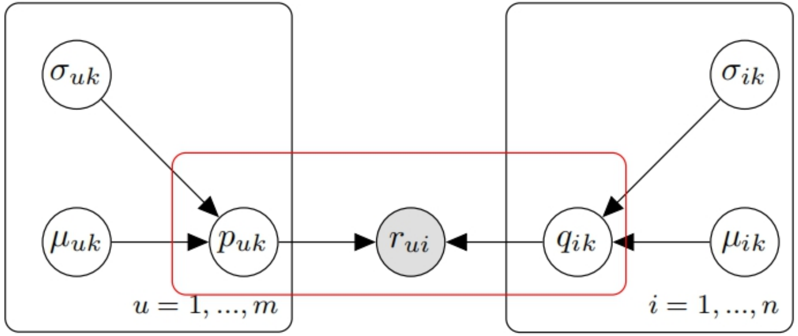

Fig. 1. The graphical model of BLFM

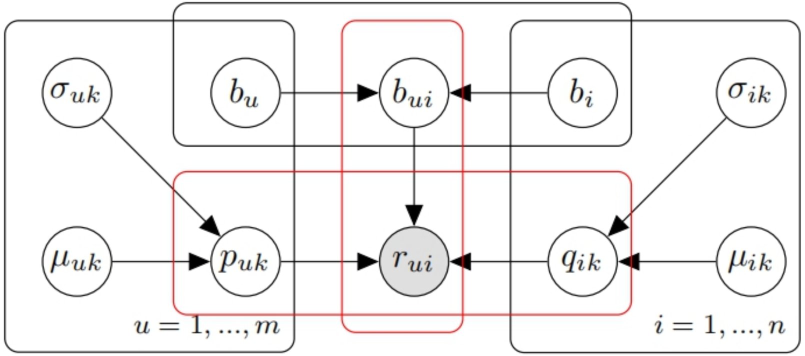

Fig. 2. The graphical model of BLFMBias.

Methodology(2)

$$q^* = \underset{q(\mathbf{p}_u, \mathbf{q}_i) \in Q}{\arg\min} \; KL(q(\mathbf{p}_u, \mathbf{q}_i) \| p(\mathbf{p}_u, \mathbf{q}_i | \mathbf{D})).$$

$$KL(q(\mathbf{p}_u, \mathbf{q}_i) \| p(\mathbf{p}_u, \mathbf{q}_i | \mathbf{D})) =$$

$$\mathbb{E}[\log q(\mathbf{p}_u, \mathbf{q}_i)] - \mathbb{E}[\log p(\mathbf{p}_u, \mathbf{q}_i | \mathbf{D})].$$

$$ELBO(q) = \mathbb{E}[\log p(\mathbf{p}_u, \mathbf{q}_i, \mathbf{D})] - \mathbb{E}[\log q(\mathbf{p}_u, \mathbf{q}_i)].$$

$$\hat{r}_{ui} = \int_{\mathbf{p}_u, \mathbf{q}_i} p(r_{ui} | \mathbf{p}_u, \mathbf{q}_i) p(\mathbf{p}_u, \mathbf{q}_i | \mathbf{D}) \mathrm{d}\{\mathbf{p}_u, \mathbf{q}_i\}.$$

Experiment(1)

Movielens				
Factors	**SVD**	**PMF**	**BPMF**	***BLFM***
RMSE@8	1.0274	1.0256	0.9871	0.9781
RMSE@16	1.0256	1.0076	0.9840	0.9836
RMSE@32	1.0278	0.9999	0.9875	0.9852
RMSE@64	1.0141	0.9952	0.9867	0.9826

Movielens			
Factors	**BLFM**	**SVDBias**	***BLFMBias***
RMSE@8	0.9781	0.9581	0.9406
RMSE@16	0.9836	0.9570	0.9397
RMSE@32	0.9852	0.9573	0.9410
RMSE@64	0.9826	0.9572	0.9452

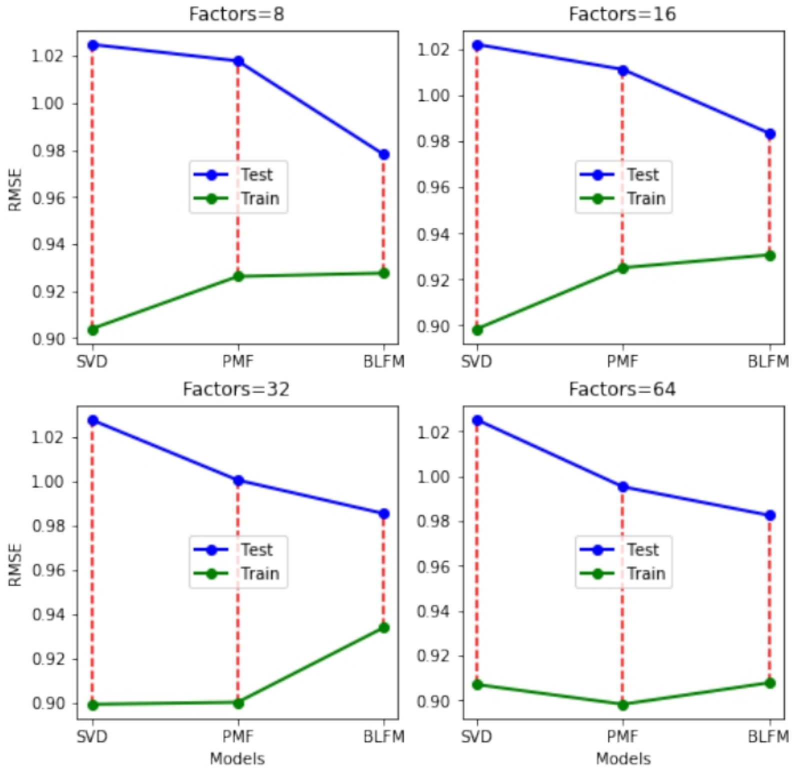

Fig. 3. Performance of RMSE on validation set and test set.

Experiment(2)

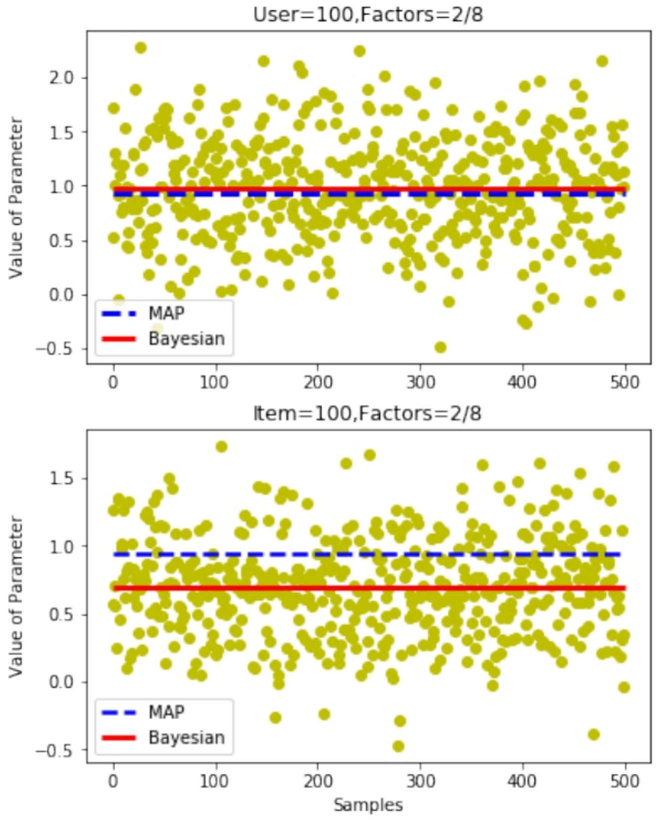

Fig. 4. Estimation of Bayesian and MAP.

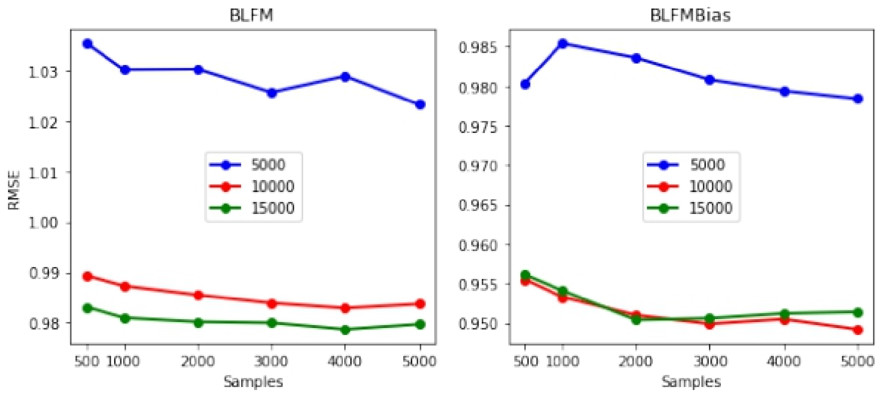

Fig. 5. Setting the number of fit iterations and samples.

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Index	**SVD**	**PMF**	**BPMF**	***BLFM***
PM	MAP	MAP	Bayesian	*Bayesian*
RG	Parameter	Parameter	Prior	*Prior*
RP	medium	lower	higher	*medium*
FS	Good	Bad	Bad	*Good*

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THANKS!

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Github: https://github.com/fjssharpsword

Blog: https://blog.csdn.net/fjssharpsword