# GRAPH SIGNAL ACTIVE CONTOURS


**Olivier LÉZORAY**

Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC, Caen, FRANCE

`olivier.lezoray@unicaen.fr`
`https://lezoray.users.greyc.fr`


GREYC Electronics and Computer Science Laboratory

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# Outline

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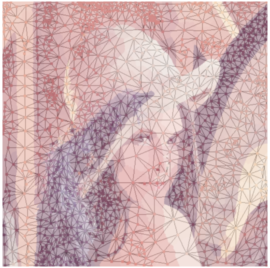
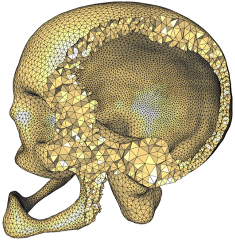
# Introduction

## Graph signals

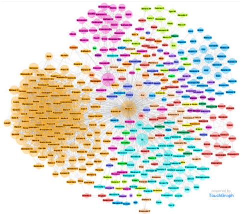
- A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists in a finite set $\mathcal{V} = \{v_1, \ldots, v_m\}$ of vertices and a finite set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ of edges.
- A graph signal $f \in \mathcal{H}(\mathcal{V})$ is defined as $f : \mathcal{G} \to \mathbb{R}^d$
- With the advent of Big Data, graph signals are everywhere !



Images



Meshes



Social graphs

# Introduction

## Graph signal segmentation

### Our objective

To conceive an approach that enable the segmentation any graph signal.

### Our building block : discrete calculus on graphs

We define a set of operators on graphs that provide similar definitions than for the continuous case.

Difference operator: $(d_w f)(v_i, v_j)$ Morphological difference operators: $(d_w^{\pm} f)(v_i, v_j)$
Gradient operator: $(\nabla_w f)(v_i)$ Gradient norm: $\|(\nabla_w f)(v_i)\|_p^p$
Morphological gradient norms: $\|(\nabla_w f)(v_i)\|_p^p = \|(\nabla_w^+ f)(v_i)\|_p^p + \|(\nabla_w^- f)(v_i)\|_p^p$
$p$-Laplace operator $(\Delta_{w,p} f)(v_i)$ Curvature operator: $(\kappa_w f)(v_i)$

### Our contribution

An adaptation on graphs of a model that combines the Geodesic Active Contour and the Active Contour Without Edges models.

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# Graph Signal Active contours

## Considered model

We consider geometric approaches with level sets.

### Geodesic Active Contours

An energy if associated to a curve $\mathcal{C}(p)$: $E_{GAC}(\mathcal{C}) = \int_0^1 g(I(C(p)))|\mathcal{C}'(p)|dp$

### Active Contours without edges

Considers two regions separated by a curve, and minimizes:

$E_{ACWE}(\mathcal{C}, c_1, c_2) = \mu \cdot \text{Length}(\mathcal{C}) + \nu \cdot \text{Area}(\text{inside}(\mathcal{C})) + \lambda_1 \int_{\text{inside}(\mathcal{C})} |I(x) - c_1|^2 dx + \lambda_2 \int_{\text{outside}(\mathcal{C})} |I(x) - c_2|^2 dx$

### Considered active contours: combines both

$E(\mathcal{C}, c_1, c_2) =$

$\mu \int_0^1 g(\mathcal{C}(p))|\mathcal{C}'(p)|dp + \nu \cdot \int_{\text{inside}(\mathcal{C})} g(\mathcal{C}(p))dA + \frac{\lambda_1}{d} \int_{\text{inside}(\mathcal{C})} \|I(x) - c_1\|_2^2 dx + \frac{\lambda_2}{d} \int_{\text{outside}(\mathcal{C})} \|I(x) - c_2\|_2^2 dx$

# Graph Signal Active contours

## Proposed adaptation on graphs

### Our adaptation

- We consider local patches $\mathcal{P}_\beta(f, v_j)$ on a $\beta$-hop subgraph $B_\beta(v_i)$
- We propose a specific potential function $g(v_i)$ that differentiates the most salient structures of a graph using patches comparison
- We consider average patch-based model to represent the regions (instead of vertex-based signal average)
- We express front propagation on graphs as $\frac{\delta f(v_i,t)}{\delta t} = \mathcal{F}(v_i)\|(\nabla_w f)(v_i,t)\|_p^p$ with $\mathcal{F}(v_i,t)$ a speed function.
- We propose a front propagation function that solves the considered active contours with discrete calculus: $\mathcal{F}(v_i,t) = \nu g(v_i) + \mu g(v_i)(\kappa_w f)(v_i,t) - \frac{\lambda_1}{d}\sum_{v_i} d^2(\mathcal{P}_\beta(f_I, v_i), \mathcal{P}_\beta^{c_1}(f_I)) + \frac{\lambda_2}{d}\sum_{v_i} d^2(\mathcal{P}_\beta(f_I, v_i), \mathcal{P}_\beta^{c_2}(f_I))$

# Outline

# Results

## Grid Graph Signals

(a) (b) (c) (d) (e) (f)

Figure: From left to right: (a) Original image, (b) Checkerboard initialization, (c) GSAC; $g(v_i) = 1$, $\beta = 0$, (d) $g(v_i)$, (e) GSAC; $g(v_i)$, $\beta = 0$, (f) GSAC; $g(v_i)$, $\beta = 1$.

# Results

## 3D Colores Meshes

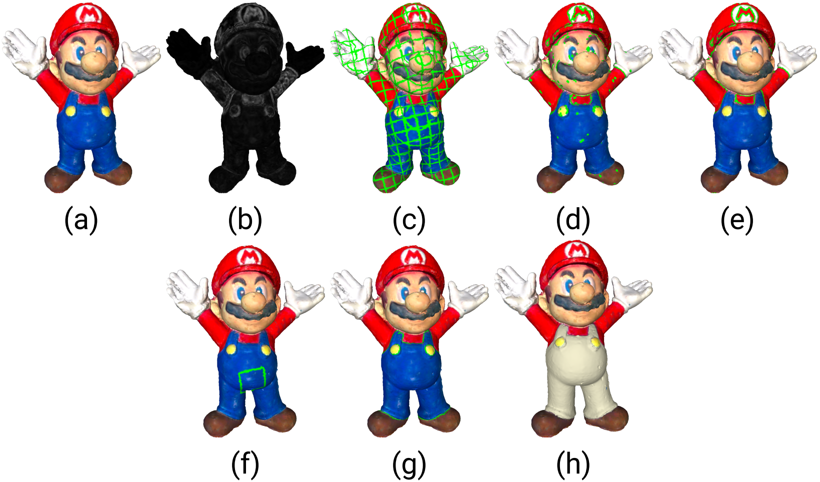

Figure: From top to bottom, left to right : (a) Original mesh, (b) $g(v_i)$ (inverted) (c) Checkerboard initialization, (d) GSAC; $g(v_i)$, $\beta = 0$, (e) GSAC; $g(v_i)$, $\beta = 2$, (f) manual initialization (g) extracted region with GSAC; $g(v_i)$, $\beta = 2$, (h) re-colorisation of the extracted region.

# Results

## Image Dataset Graph

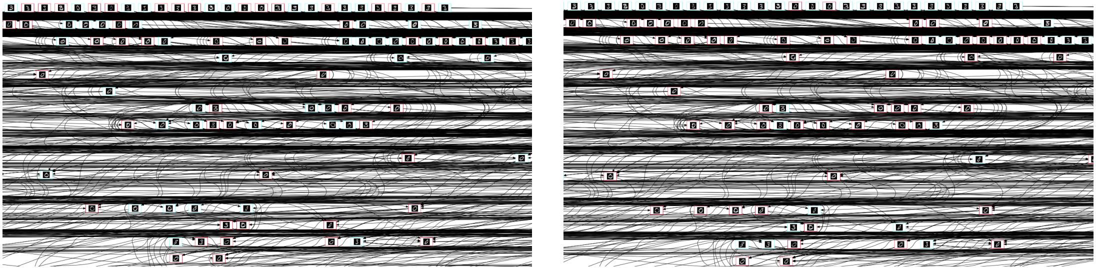

Figure: Classification of a subset (digits 0 and 3) of the MNIST dataset. The colors around each image show the class it is affected to. The top row shows the initialization and bottom second row the final classification.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 98.8 | 95.8 | 97 | 97.05 | 90.7 | 95.55 | 96.75 | 95.95 | 96.25 |

Table: Classification scores for the $0$ digit versus each other digit of the MNIST database.

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# Conclusion

## Our contribution

- Proposed an adaptation of active contours on graphs that combines the Geodesic Active Contour and Active Contours Without Edges approaches
- A level-set formulation has been adapted on graphs with a framework of graph operators that can describe the evolution of a front on a graph
- We incorporate specific graph features extracted in the form of a potential function and local graph patches to enhance the segmentation
- Presented many results on various graph signals

## Future works

- Multi-label extension using Voronoi Implicite Interface Model

# The End



Publications available at :

`https://lezoray.users.greyc.fr`

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