Double Manifolds Regularized Non-negative Matrix Factorization for Data Representation

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2021.1

ICPR2020, Milan, Italy

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Content

- Non-negative Matrix Factorization (NMF)
- Graph regularized NMF (GNMF)
- Double Manifolds Regularized Non-negative Matrix Factorization (DMR-NMF)

Previous Works: NMF

- NMF aims to find two non-negative matrices whose product can well approximate the original matrix.
- Mathematically, Given a data matrix $X = \{x_1, \cdots, x_n\} \in R^{d\times n}$, each column of X is a sample vector. NMF aims to find two nonnegative matrices $U = [u_{ij}] \in R^{d\times k}$ and $V = [v_{ij}] \in R^{k\times n}$ which minimize the following objective function:

$$min_{U\geq 0, V\geq 0} \quad \|X - UV\|_F^2$$

D. D. Lee and H. H. Seung. Learning the parts of objects by non-negative matrix factorization. Nature 1999

Previous Works: NMF

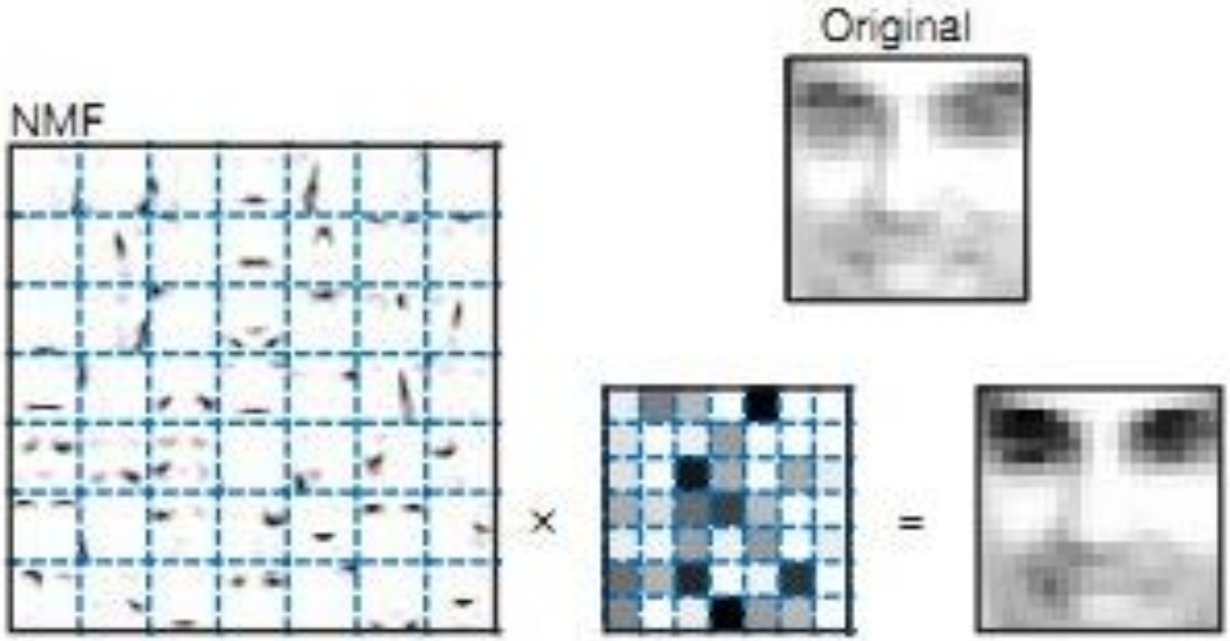
Optimization: iterative updates of the two factor matrices U, V

$$u_{ij}^{t+1} \leftarrow u_{ij}^{t} \frac{(XV_t^T)_{ij}}{(U_t V_t V_t^T)_{ij}}$$

$$v_{ij}^{t+1} \leftarrow v_{ij}^{t} \frac{(U_{t+1}^T X)_{ij}}{(U_{t+1}^T U_{t+1} V_t)_{ij}}$$

Convergence: above update steps will find a local minimum of the objective function.

Previous Works: NMF



Previous Works: GNMF

- NMF performs this learning in the Euclidean space. It fails to discover the intrinsic geometrical and discriminating structure of the data space, which is essential to the real applications.

- Graph regularized Non-negative Matrix Factorization (GNMF) algorithm avoids this limitation by incorporating a geometrically based regularizer.

Previous Works: GNMF

- Graph Laplacian regularizer

$$\frac{1}{2}\sum_{i,j=1}^{n} w_{ij}\left\|v_i - v_j\right\|^2$$

It can be rewritten as

$$Tr(VLV^T)$$

where $L = D - W$ is the graph Laplacian matrix of adjacency matrix W and D is the diagonal matrix W.

Previous Works: GNMF

- the graph regularized NMF (GNMF) model is formulated as

$$min_{U\geq 0,V\geq 0} \quad \|X - UV\|_F^2 + \alpha Tr(VLV^T),$$

where $L = D - W$ is the graph Laplacian matrix of adjacency matrix W and D is the diagonal matrix W.

D. Cai, X. He, J. Han and T. S. Huang. Graph regularized nonnegative matrix factorization for data representation. PAMI 2010

DMR-NMF

Motivation:

- The similarity matrix W is artificially defined according to raw feature in advance, which may be not accurate since the existence of noises. Thus, W in GNMF is not an optimal graph for characterizing the complex intrinsic structure of data.
- And, the global structure of data is not explored for GNMF. Those reduce the flexibility of NMF and heavily affect the performance of the algorithm.

DMR-NMF

- Low rank representation(LRR) aims to seek low-rank affinity graph which can effectively reveal the global structures of data.
- And due to the complex data distribution, the single manifold structures (such as only global or only local) may be not sufficient to describe the underlying true structure.

G. Liu, Z. Lin, J. Sun, Y. Yu and Y. Ma. Robust Recovery of Subspace Structures by Low Rank Representation. PAMI 2013.

DMR-NMF

The objective function of DMR-NMF is formulated as

$$\mathcal{O} = \|X - UV\|_F^2 + \alpha Tr(VLV^T) + \beta \|Z\|_* + \gamma \|V - VZ\|_F^2.$$

Similar to NMF and GNMF, we add the non-negative constraints on U, V, i.e.,

$$min_{U \geq 0, V \geq 0, Z} \ \mathcal{O}(X, U, V, L, Z)$$

Experiments

Data set	K	ACC					NMI				
		Kmeans	NMF	GNMF	SDNMF	DMR-NMF	Kmeans	NMF	GNMF	SDNMF	DMR-NMF
ORL	10	75.80	75.80	77.60	77.70	**80.00**	73.78	74.36	74.69	72.34	**78.62**
	20	63.70	66.70	68.20	68.50	**69.30**	72.34	72.58	74.85	75.54	**76.08**
	30	59.33	61.40	61.00	61.58	**63.17**	70.62	71.32	72.73	71.06	**73.19**
	40	57.50	57.30	58.95	**60.90**	59.75	71.50	71.62	72.44	72.40	**73.62**
UMIST	5	76.20	79.80	89.40	89.00	**92.40**	68.42	70.22	84.77	83.54	**87.03**
	10	66.70	70.00	80.00	79.10	**83.00**	67.37	69.09	78.75	78.26	**80.90**
	15	61.47	60.40	68.87	69.60	**72.27**	66.70	65.27	73.78	74.58	**76.14**
	20	54.65	56.95	66.75	64.60	**68.55**	66.66	67.20	72.81	71.79	**74.90**
Yale	5	79.27	78.18	84.00	78.55	**86.18**	64.50	67.34	73.84	64.53	**74.08**
	7	76.36	74.29	78.44	76.10	**82.86**	58.62	60.54	64.74	64.50	**68.47**
	9	68.48	66.46	71.52	71.11	**73.94**	54.97	54.02	60.70	55.60	**61.83**
	11	60.99	61.16	63.14	61.36	**65.95**	49.37	50.74	54.22	49.83	**55.31**
	13	58.18	57.90	58.80	57.48	**60.35**	49.12	48.74	49.56	51.04	**52.31**
	15	55.88	54.52	56.36	57.58	**58.42**	49.65	48.34	51.86	51.73	**52.04**
COIL20	4	91.32	90.90	95.35	95.42	**98.71**	84.07	80.94	92.01	92.01	**95.32**
	8	85.14	85.66	91.70	91.15	**92.88**	79.73	78.75	88.84	85.70	**90.09**
	12	78.45	73.52	85.32	86.92	**88.23**	76.68	72.09	81.28	84.73	**85.28**
	16	73.47	69.64	81.25	80.94	**83.09**	72.81	68.43	82.73	83.56	**84.23**
	20	68.56	66.90	76.25	78.54	**79.55**	71.55	67.42	80.69	81.41	**81.43**

Thanks!