Low Rank Representation on Product Grassmann Manifolds for Multi-view Subspace Clustering

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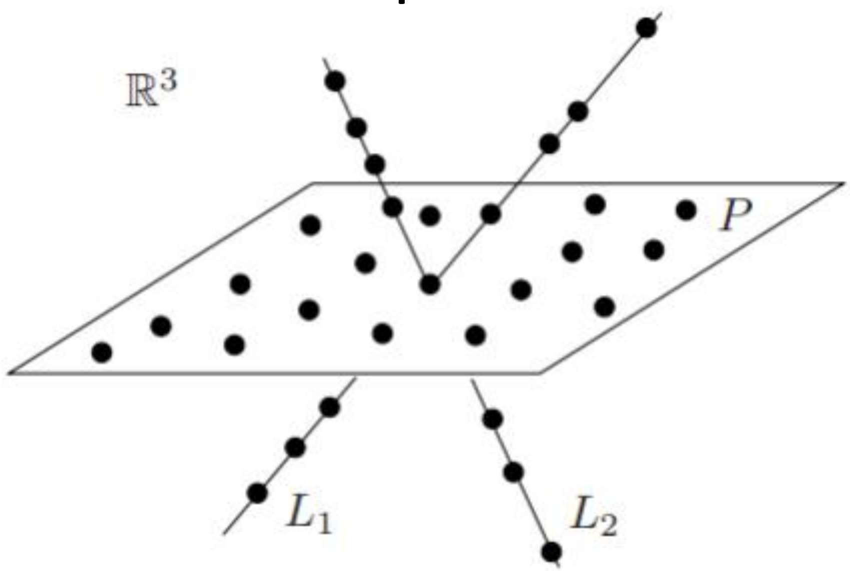
Subspace clustering

- Given sufficient data samples drawn from multiple low dimensional subspaces:

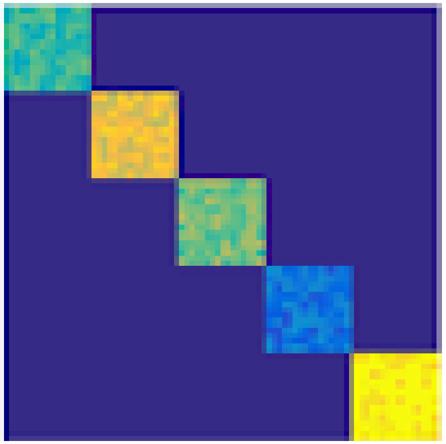
the **goal** is to group a set of data samples into several clusters, which clusters corresponds to the **independent** subspaces.



Algorithms

Spectral Clustering (two steps):

- Graph construction : construct a graph (i.e., affinity matrix) to measure the similarities between data samples;



- Spectral clustering algorithm group the data samples into multiple clusters.

Related Works: LRR

The basic assumption: data points are sampled from a union of k independent subspaces.

- Graph construction by low rank representation

$$min_Z \; rank(Z) \quad s.t. \; X = XZ$$

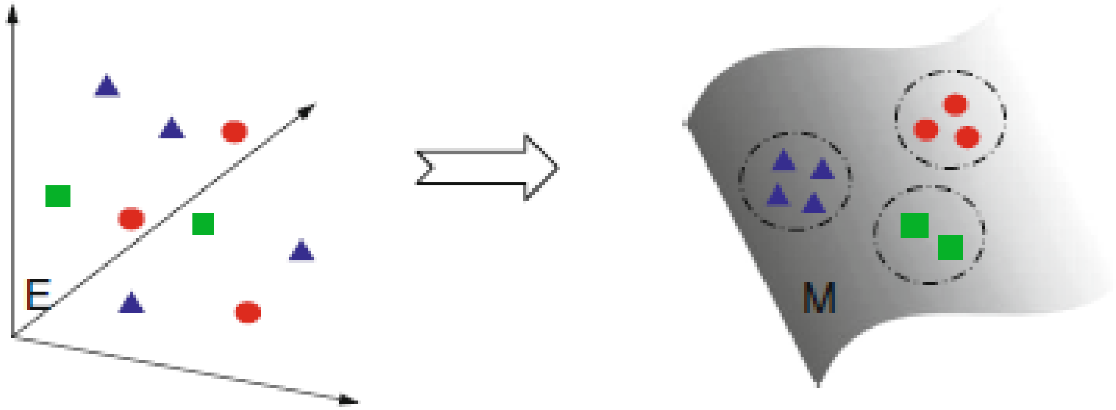
$$min_Z \; \|Z\|_* \quad s.t. \; X = XZ$$

$$min_{Z,E} \; \|Z\|_* + \alpha \|E\|_{2,1} \quad s.t. \; X = XZ + E$$

- Graph matrix $W = \frac{|Z|^T + |Z|}{2}$.

Related Works: G-LRR

the high-dimension data in general lie in or close to a low dimensional manifold. Thus, extending the LRR based methods on the Grassmann manifold for high-dimension data clustering with the non-Euclidean geometry.



Euclidean Space **Grassmann Manifold**

Background: Product Grassmann Manifolds

- Grassmann manifolds, denoted by $\mathcal{G}(p,d)$ is the space of all p-dimensional linear subspaces of for $R^d (0 \leq p \leq d)$.
- Grassmann manifolds can be embedded into the space of symmetric matrices $Sym(d)$ as

$$\pi\colon \mathcal{G}(p,d) \to Sym(d), \qquad \pi(X) = XX^T.$$

- Replace the distance on Grassmann manifolds with the following distance defined on the symmetric matrix space

$$d_{\mathcal{G}}^2(X,Y) = \frac{1}{2}\|\pi(X) - \pi(Y)\|_F^2$$

Related Works: G-LRR

For a given sample set $\mathbb{X} = \{X_1, X_2, \cdots, X_n\}$ where $X_i \in \mathcal{G}(p, d)$, the **G-LRR** is formulated as :

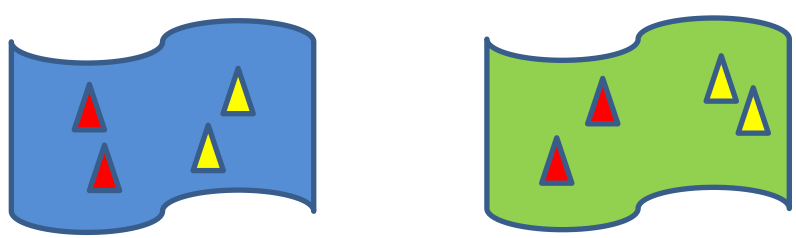
$$min_Z \sum_{i=1}^{n} \left\| X_i \ominus (\uplus_{j=1}^{n} z_{ij} \odot X_j) \right\|_{\mathcal{G}} + \alpha \|Z\|_*$$

where abstract symbols $\ominus$, $\uplus_{j=1}^{n}$, $\odot$denote the ``linear'' operations to be defined on manifolds, i.e., addition, subtraction and scalar multiplication. $\left\| X_i \ominus (\uplus_{j=1}^{n} z_{ij} \odot X_j) \right\|_{\mathcal{G}}$ with operator $\ominus$ representing the product manifold distance between X_i and its reconstruction $\uplus_{j=1}^{n} z_{ij} \odot X_j$.

B. Wang, Y. Hu, J. Gao, Y. Sun and B. Yin. Low Rank Representation on Grassmann Manifolds. In ACCV 2014

Related Works: PG-LRR

- Given V Grassmann manifolds with dimensions $p_1, \cdots, p_V$ respectively, the Product Grassmann manifolds (PGM) (denoted by $\mathcal{PG}(d; p_1, \cdots, p_V)$) is defined as $\mathcal{G}(p_1, d) \times \cdots \times \mathcal{G}(p_V, d)$.
- Then, a point embedded in PGM is a set of Grassmann points, denoted by $[X] = \{X^1, \cdots X^V\}$ where $X^i \in \mathcal{G}(p_i, d)$.



$\mathcal{G}(p_1, d)$ $\mathcal{G}(p_2, d)$

Related Works: PG-LRR

- A valid distance on PGM can be induced from the individual distance on each Grassmann manifold as follows

$$d_{\mathcal{PG}}^2(X, Y) = \sum_{v=1}^{V} d_{\mathcal{G}}^2(X^v, Y^v)$$

Related Works: PG-LRR

$\mathcal{X} = \{[X_1], \cdots, [X_n]\}$be a set of given PGM samples, where$[X_i] = \{X_i^1, \cdots, X_i^V\} \in \mathcal{PG}(d; p_1, \cdots, p_V)$ with the basic matrix $X_i^v \in \mathcal{G}(p_v, d)$. Then, PG-LRR is formulated as

$$min_Z \sum_{i=1}^{n} \left\| [X_i] \ominus (\uplus_{j=1}^{n} z_{ij} \odot [X_j]) \right\|_{\mathcal{PG}} + \alpha \|Z\|_*$$

Remark: the objective of PG-LRR is convex of Z.

B. Wang, Y. Hu, J. Gao, Y. Sun and B. Yin, Product grassman manifold representation and its lrr models. In AAAI, 2016

PG-MFLRR

Motivation: LRR, G-LRR, and PG-LRR directly employ convex nuclear norm $\|Z\|_*$ to approximate low rank constraint rank(Z), which may be a biased estimation of the rank. Despite the elegant theory of the convex relaxation of rank(Z),it has two major weaknesses:

1) over-relaxation of rank components leads to the results which can be far from the true underlying ones;
2) the singular value decomposition (SVD) of matrix has high complexity in computation.

PG-MFLRR

$$min_Z \sum_{i=1}^{n} \left\| [X_i] \ominus (\uplus_{j=1}^{n} z_{ij} \odot [X_j]) \right\|_{\mathcal{PG}} + \alpha \|M\|_*$$

$$s.t. \;\; Z = UMV^T, U^T U = V^T V = I_k,$$

Where the fixed-rank $k \ll n$ is a reasonable assumption which provides the approximate rather than random upper bound for true rank of Z, resulting in a more accurate representation.

Theoretical analysis

***Theorem*. ** A global minimum of non-convex model PG-MFLRR can always be obtained.

Proof: we can construct the optimal solution skillfully according to the optimum of convex model PG-LRR.

Computational complexity analysis

PG-LRR: $\mathcal{O}(\mathrm{t}(2\mathrm{n}^3))$

PG-MFLRR: $\mathcal{O}(t(n^3 + 3n^2k))$

Experiments: Multi-video clustering

	G-LRR	**MVGL**	**MCGC**	**SM2SC**	**LCRSR**	**PG-LRR**	**PG-MFLRR**
ACC	0.4541	0.1269	0.1429	0.1463	0.3766	0.4957	**0.5098**
NMI	0.5421	0.0492	0.0649	0.0628	0.2397	0.6250	**0.6421**
F-score	0.1296	0.1298	0.1445	0.0782	0.2217	0.5102	**0.5317**

Table 1. Custering results on **ACT4** video database

Experiments: Multi-video clustering

	G-LRR	**MVGL**	**MCGC**	**SM2SC**	**LCRSR**	**PG-LRR**	**PG-MFLRR**
ACC	0.2904	0.2775	0.2679	0.1100	0.2700	0.2969	**0.3560**
NMI	0.2202	0.2024	0.1972	0.0042	0.2078	0.3525	**0.3681**
F-score	0.2976	0.2932	0.2710	0.1798	0.0641	0.3017	**0.3978**

Table 1. Custering results on **NUCLA** video database

Experiments: Multi-video clustering

	G-LRR	MVGL	MCGC	SM2SC	LCRSR	PG-LRR	PG-MFLRR
ACC	0.4196	0.4041	0.4071	0.3835	0.3890	0.4240	**0.4945**
NMI	0.4669	0.4831	0.4448	0.4095	0.3740	0.4773	**0.5014**
F-score	0.4187	0.4289	0.2710	0.2808	0.3889	0.4268	**0.4912**

Table 1. Custering results on **IXMAS** video database

Experiments: Multi-video clustering

	G-LRR	**MVGL**	**MCGC**	**SM2SC**	**LCRSR**	**PG-LRR**	**PG-MFLRR**
ACC	0.7802	0.7802	0.9396	0.7637	0.8381	0.8022	**1.0000**
NMI	0.6870	0.6870	0.8436	0.5275	0.7237	0.6075	**1.0000**
F-score	0.7059	0.7913	0.9393	0.6399	0.7734	0.8014	**1.0000**

Table 1. Custering results on **DTHC** video database

Thanks!