Minimal Solvers for Indoor UAV Positioning

Marcus Valtonen Örnhag[1], Patrik Persson[1], Mårten Wadenbäck[2], Kalle Åström[1] and Anders Heyden[1]

International Conference on Pattern Recognition, 2020

[1]Centre for Mathematical Sciences, Lund University
[2]Department of Electrical Engineering, Linköping University

Minimal Solvers

Why minimal solvers? They

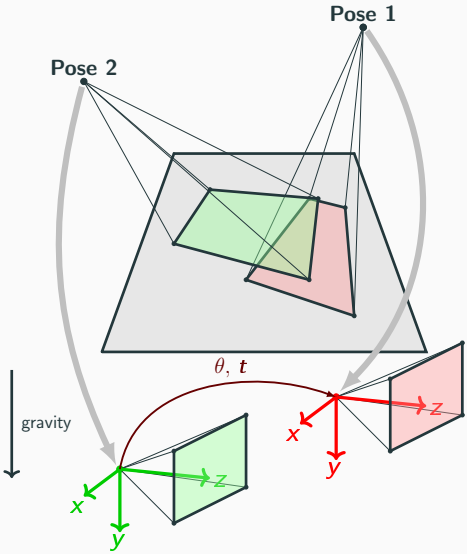
- intrinsically encode the geometric properties one wishes to impose on the solution,
- require fewer iterations in robust estimation frameworks, such as RANSAC.

Problem Geometry

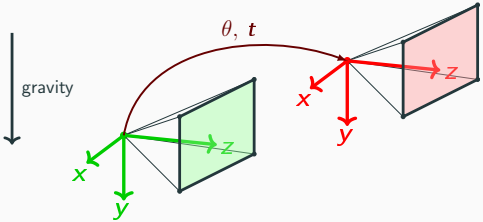
Camera matrices

$$\boldsymbol{P}_1 = \boldsymbol{K}_1 \left[\boldsymbol{R}_{\text{imu}}^{(1)} \quad | \, \boldsymbol{0} \right]$$

$$\boldsymbol{P}_2 = \boldsymbol{K}_2 \left[\boldsymbol{R}_{\text{imu}}^{(2)} \boldsymbol{R}_y \mid \boldsymbol{t} \right]$$



Constructing the *fHf* solver

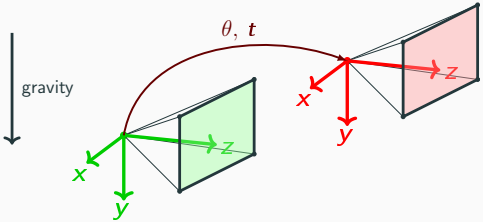


Homography (normalized)

$$\boldsymbol{H}_y \sim \boldsymbol{R}_y + \boldsymbol{t}\boldsymbol{n}^T$$

where $\boldsymbol{n} = [0,\ 0,\ 1]^T$ is a floor normal.

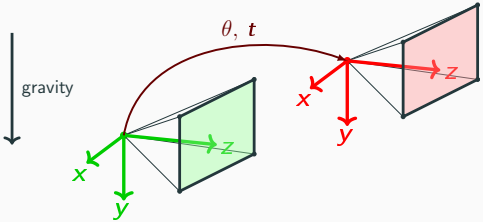
Constructing the *fHf* solver



Relation to general homography $\boldsymbol{H}$

$$\boldsymbol{H}_y \sim \boldsymbol{R}_{\text{imu}}^{(2)T} \boldsymbol{K}_2^{-1} \boldsymbol{H} \boldsymbol{K}_1 \boldsymbol{R}_{\text{imu}}^{(1)}$$

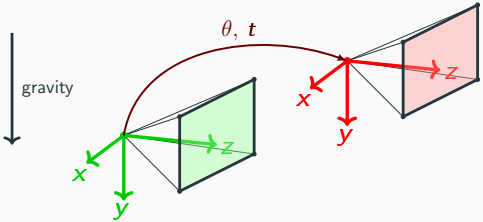
Constructing the *fHf* solver



The DLT relation

$$\boldsymbol{x}_2 \sim \boldsymbol{H}\boldsymbol{x}_1$$

Constructing the *fHf* solver



Five unknowns:

- Translation (3 DoF)
- Rotation about y-axis (1 DoF)
- Focal length (1 DoF)

Minimal case requires 2.5 points.

Constructing the fHf solver

We may parameterize the homography as

$$\boldsymbol{H}_y = \begin{bmatrix} h_1 & h_3 & h_2 \\ 0 & h_4 & 0 \\ -h_2 & h_5 & h_1 \end{bmatrix},$$

and enforce the Pythagorean trigonometric identity

$$h_1^2 + h_2^2 = 1 \ .$$

Constructing the *fHf* solver

The invserse calibration matrix

$$\boldsymbol{K}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & w \end{bmatrix},$$

where $w \neq 0$.

Constructing the *fHf* solver

The DLT equations now contain six unknowns, but we have added an extra quadratic constraint $h_1^2 + h_2^2 = 1$. This results in a system

$$\boldsymbol{M}\boldsymbol{v} = \boldsymbol{0},$$

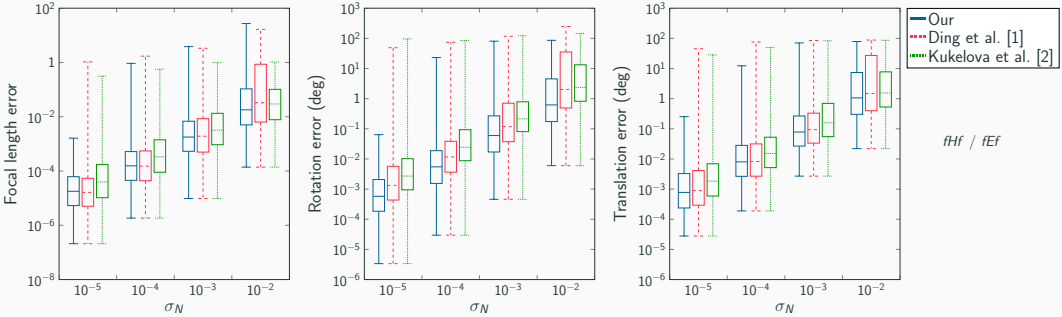
where $\boldsymbol{M} \in \mathbb{R}^{6\times 18}$ and $\boldsymbol{v} \in \mathbb{R}^{18}$ is the vector of monomials.

Constructing the *fHf* solver

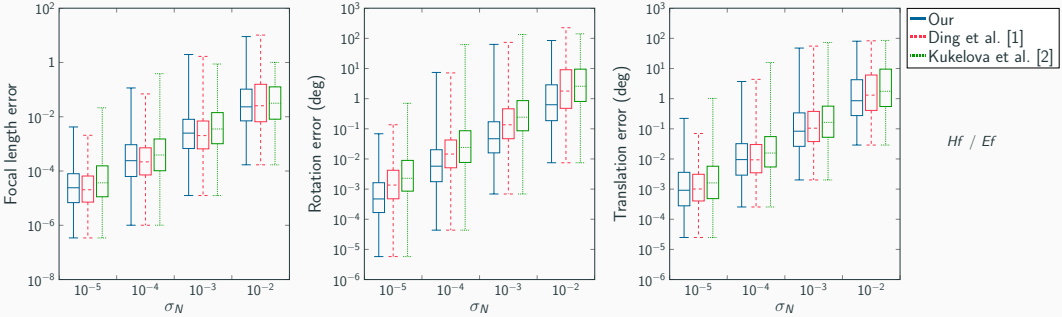
There are in general 14 solutions (when enforcing $w \neq 0$). Utilizing symmetry, we only need to solve for seven of these.

The elimination template is of size 10×17.

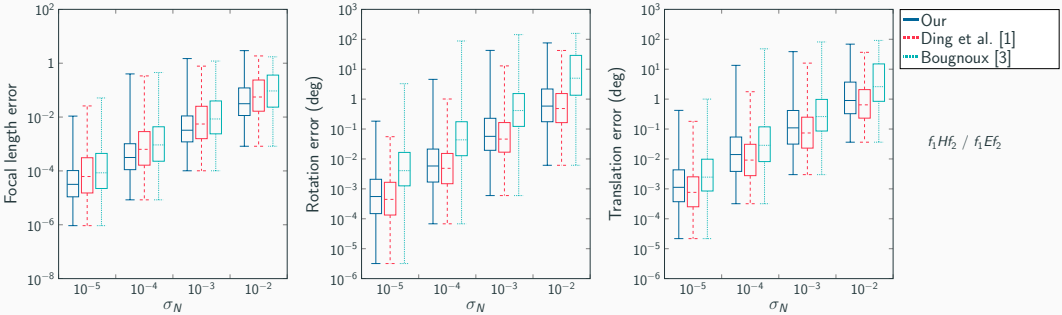
Noise sensitivity



Noise sensitivity



Noise sensitivity



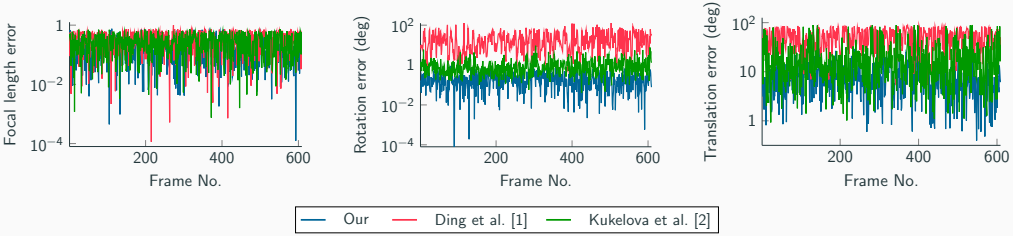
Timings

Mean execution time for 10,000 randomly generated problems in C++.

Case	Author	Execution time (μs)
fHf	Our	14
	Ding *et al.* [1]	1052
fEf	Kukelova *et al.* [2]	103
Hf	Our	5
	Ding *et al.* [1]	124
Ef	Kukelova *et al.* [2]	25
f_1Hf_2	Our	9
	Ding *et al.* [1]	47
f_1Ef_2	Bougnoux [3]	27

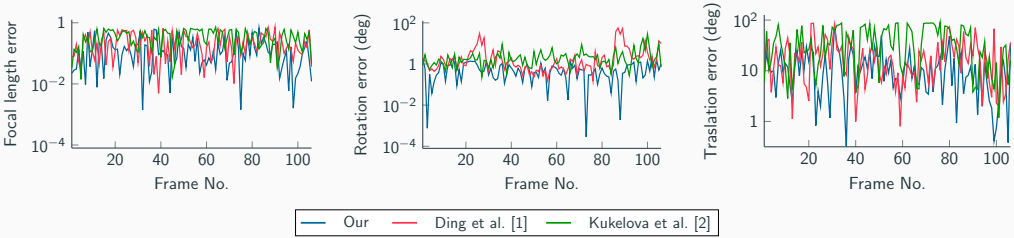
Real experiments with UAVs

Basement (610 frames, 2978 keypoints)



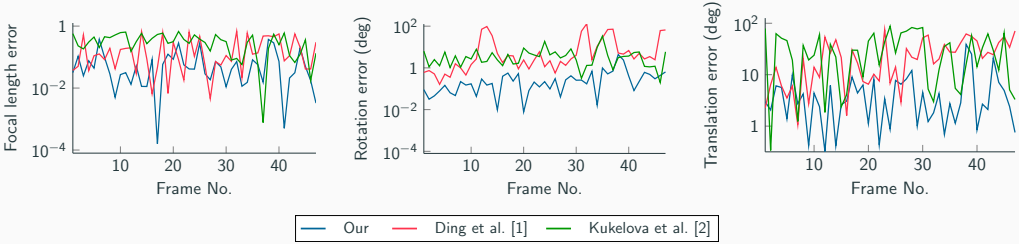
Real experiments with UAVs

Carpet (107 frames, 400 keypoints)



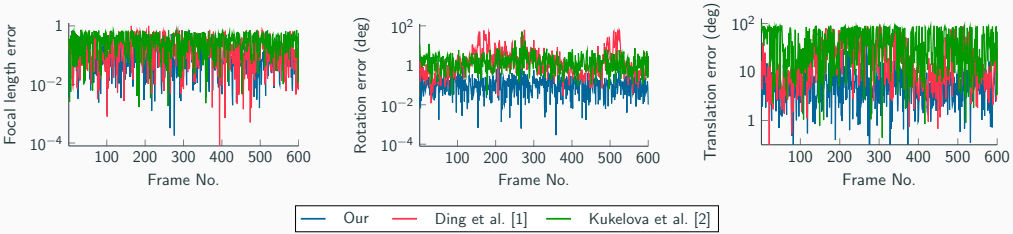
Real experiments with UAVs

Indoor (48 frames, 184 keypoints)



Real experiments with UAVs

Outdoor (601 frames, 3659 keypoints)



Thank you for watching!

Check out our code on Github!

https://github.com/marcusvaltonen/HomLib

References

[1] Y. Ding, J. Yang, J. Ponce, and H. Kong, "An efficient solution to the homography-based relative pose problem with a common reference direction," in *The IEEE International Conference on Computer Vision (ICCV)*, October 2019.

[2] Z. Kukelova, J. Kileel, B. Sturmfels, and T. Pajdla, "A clever elimination strategy for efficient minimal solvers," *Conference on Computer Vision and Pattern Recognition (CVPR)*, pp. 3605–3614, 2017.

[3] S. Bougnoux, "From projective to euclidean space under any practical situation, a criticism of self-calibration," in *International Conference on Computer Vision (ICCV)*, 1998, pp. 790–798.