3D Pots Configuration System by Optimizing over Geometric Constraints

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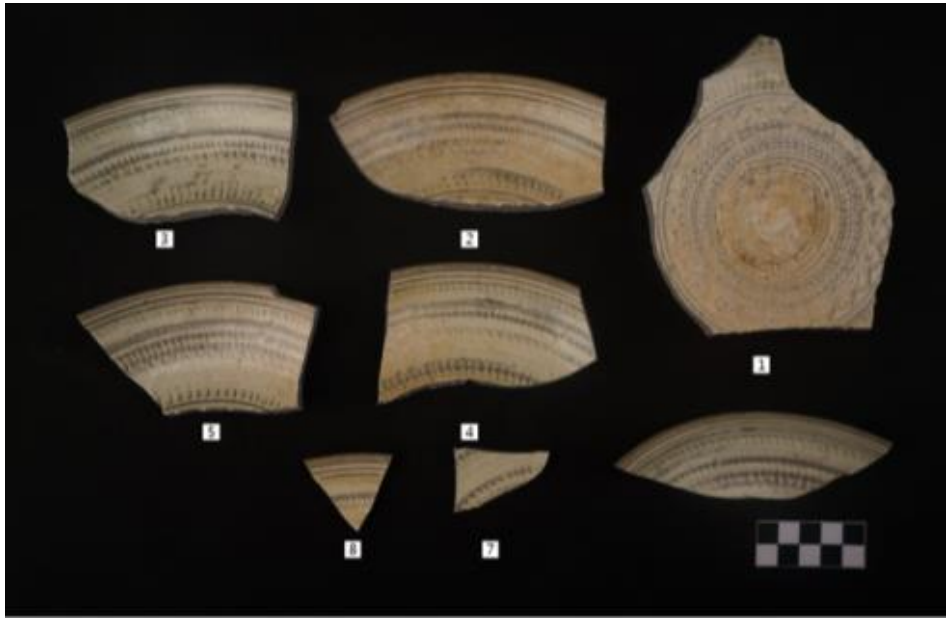
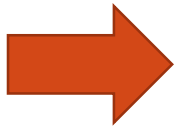
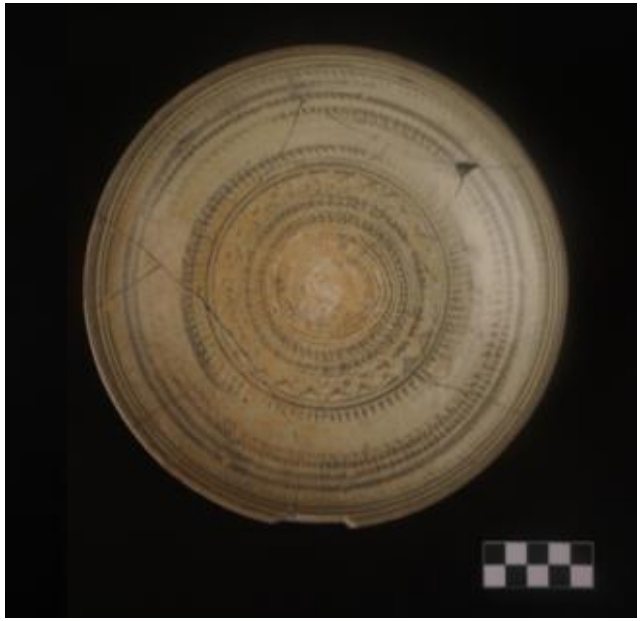
Motivation

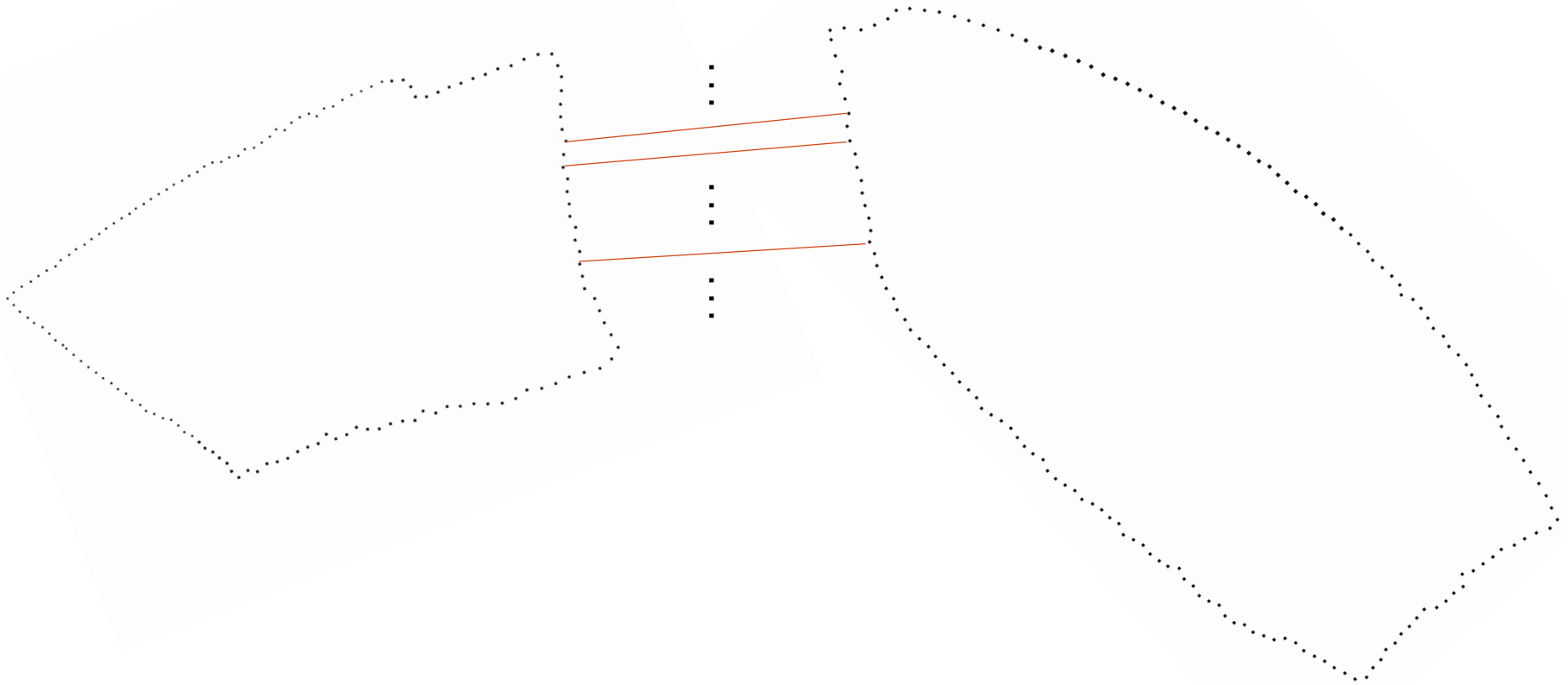






Problem Formulation

Find a binary matrix B encoding match between sampled points on the pottery fragments' break lines.



Preprocessing – Building a Matching Score Matrix M

- $M(m^{(i_1)}, m^{(i_2)})_{j_1 j_2}$ = (probabilistic score of match between the j_1^{th} point of fragment i_1 and the j_2^{th} point of fragment i_2)
- The probabilistic matching scores are calculated based on the similarity between the points' feature vectors.

$$M = \begin{pmatrix} M(\mathbf{m}^{(1)}, \mathbf{m}^{(1)}) & \cdots & M(\mathbf{m}^{(1)}, \mathbf{m}^{(N)}) \\ \vdots & \ddots & \vdots \\ M(\mathbf{m}^{(N)}, \mathbf{m}^{(1)}) & \cdots & M(\mathbf{m}^{(N)}, \mathbf{m}^{(N)}) \end{pmatrix}$$

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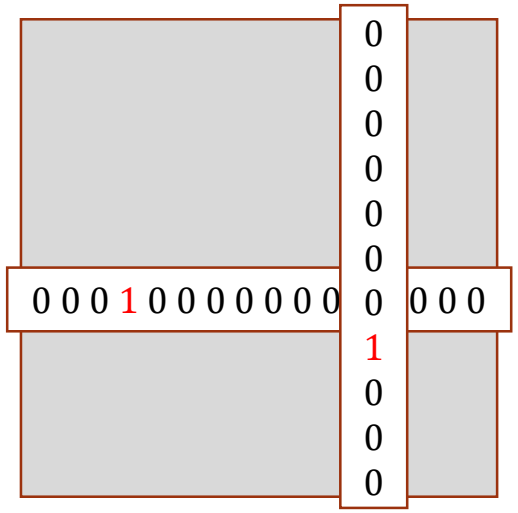
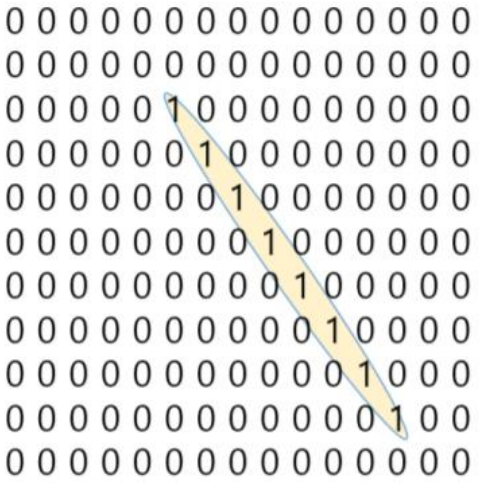
Optimization

- From M, we want to find a binary matrix B where matched break lines are encoded as 1 and other elements are 0.
- In addition to being close to M, B must satisfy the following 2 constraints :

1. Bijection



2. Continuity



Optimization

- The problem can be formulated into a binary optimization problem, which can be relaxed into convex quadratic programming.

Binary Optimization Problem

$$\arg\min_{X} \quad \|X - M\|_F^2 + \sum_{i,j} s_i s_j \|X_{ij} - X_{(i+1)(j+1)}\|_2^2$$

$$\text{subject to } X^\top \mathbf{1} \leq \mathbf{1},\ X\mathbf{1} \leq \mathbf{1},$$
$$X_{ij} \in \{0, 1\}, \forall i, j.$$

Relaxed Convex Constraints

$$X^\top \mathbf{1} \leq \mathbf{1},\ X\mathbf{1} \leq \mathbf{1},\ 0 \leq X$$

(s_i and s_j are binary variables indicating whether continuity should be enforced or not.)

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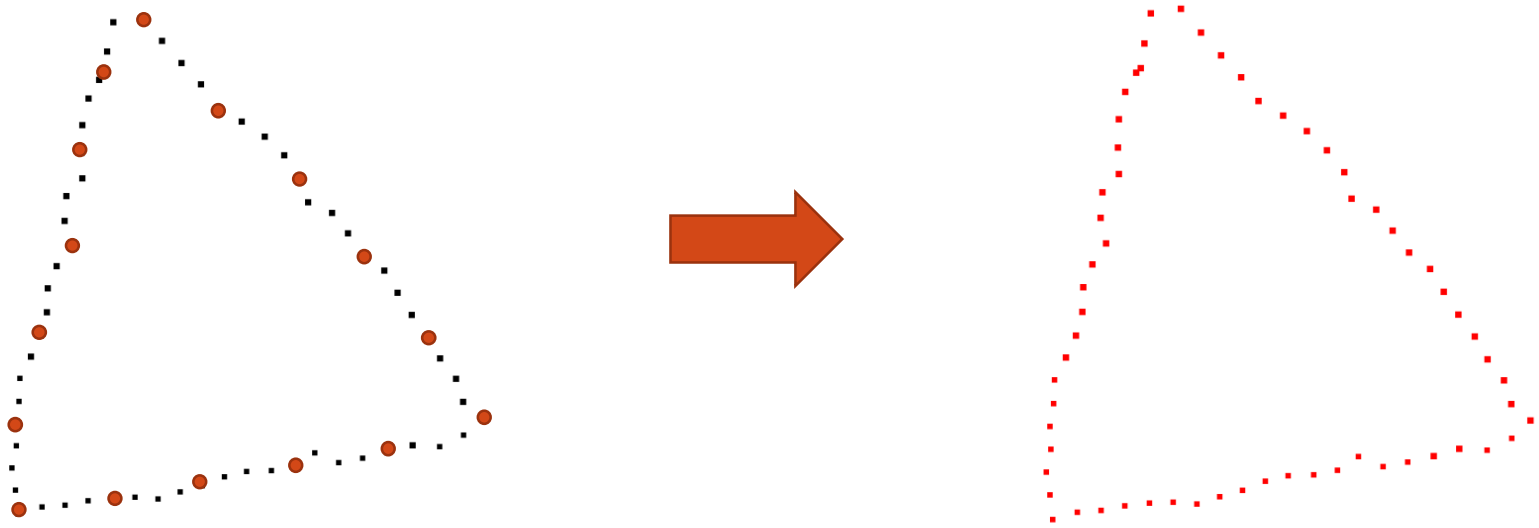
Adaptation to Large-Scale Data
- Hierarchical Extension

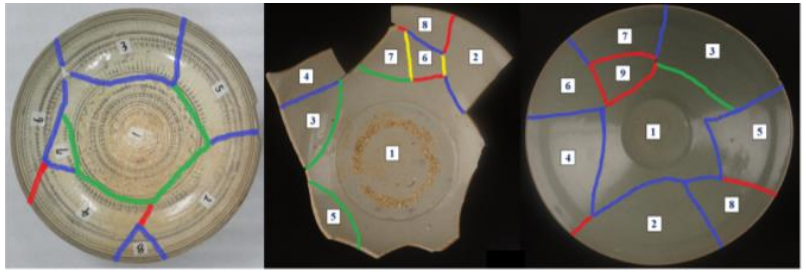


Experiment Results on Real Pottery Data

- Ablation :
  - With just feature matching (i.e. without the optimization process), hundreds of false-positives were reported between non-adjacent fragments alone when attempting to reassemble the 3 potteries altogether.

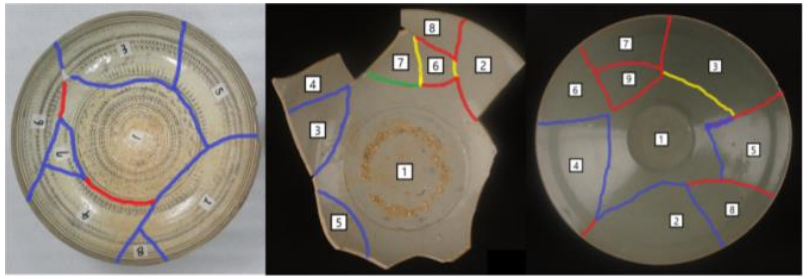
Color Legends Indicating Success of Match

color	match detected	correct location	wrong location
—	×	-	-
—	✓	×	✓
—	✓	✓	✓
—	✓	✓	×



(3 False-Positives)

Result of Our Algorithm Run Separately on Each Pottery



(10 False-Positives)

Result of the Hierarchical Extension of Our Algorithm on the Mixture of the Fragments from the Three Potteries

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Conclusion

- Effective - finds the correct configuration of pottery fragments with extracted shape features along the break lines.
- Simple - employing only convex quadratic programming.
- Flexible - with no restriction on how to define the feature vector.