Feature Extraction by Joint Robust Discriminant Analysis and Inter-class Sparsity

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Overview

- **Introduction and Motivations**
- Proposed Method
- Experiments and Results
- Conclusion

Motivations

- Discriminant method aiming to learn a discriminative projection space for the data.
- Better data representation.
- Find a mapping that retains several interesting properties for learning tasks

Related works

- Linear Discriminant Analysis (LDA)
- Principal Component Analysis
- inter-class Sparsity based discriminative least square regression (ICS DLSR) [Wen et al.,2018]
- Robust sparse LDA (RSLDA) [Wen, Fang et al., 2018]

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Properties

- Supervised Learning Method
- Robust and Discriminant
- The model provides feature ranking
- Provides Inter-class sparsity

Classical LDA

Fisher Criterion:

$$\mathbf{p} = \arg\max_{\mathbf{p}} \frac{\mathbf{p}^T \mathbf{S}_b \mathbf{p}}{\mathbf{p}^T \mathbf{S}_w \mathbf{p}}$$

Difference form:

$$\mathbf{p} = \arg\min_{\mathbf{p}^T\mathbf{p}=1} \mathbf{p}^T (\mathbf{S}_w - \mu\, \mathbf{S}_b)\, \mathbf{p}$$

p is the eigenvector associated with the smallest eigenvalue of (Sw − μ Sb).

$$\mathbf{S}_w = \frac{1}{N} \sum_{i=1}^{C} \sum_{j=1}^{n_i} (\mathbf{x}_j{}^i - \mu_i)(\mathbf{x}_j{}^i - \mu_i)^T$$

$$\mathbf{S}_b = \frac{1}{N} \sum_{i=1}^{C} n_i (\mu_i - \mu)(\mu_i - \mu)^T$$

Trace Form: $\boxed{Tr\ \left(\mathbf{Q}^T (\mathbf{S}_w - \mu\, \mathbf{S}_b)\, \mathbf{Q}\right)}$

μ is a small positive constant.
C is the number of classes
N_i is the number of samples of the i-th class

Proposed objective function

$$f(\mathbf{Q},\mathbf{P}) = \boxed{Tr\left(\mathbf{Q}^T\,\mathbf{S}\,\mathbf{Q}\right)} + \lambda_1 \boxed{\|\mathbf{Q}\|_{2,1}} + \lambda_2 \boxed{\|\mathbf{X}-\mathbf{P}\,\mathbf{Q}^T\,\mathbf{X}\|_1} + \lambda_3 \boxed{\sum_{i=1}^{C} \|\mathbf{Q}^T\,\mathbf{X}_i\|_{2,1}}$$

LDA criterion

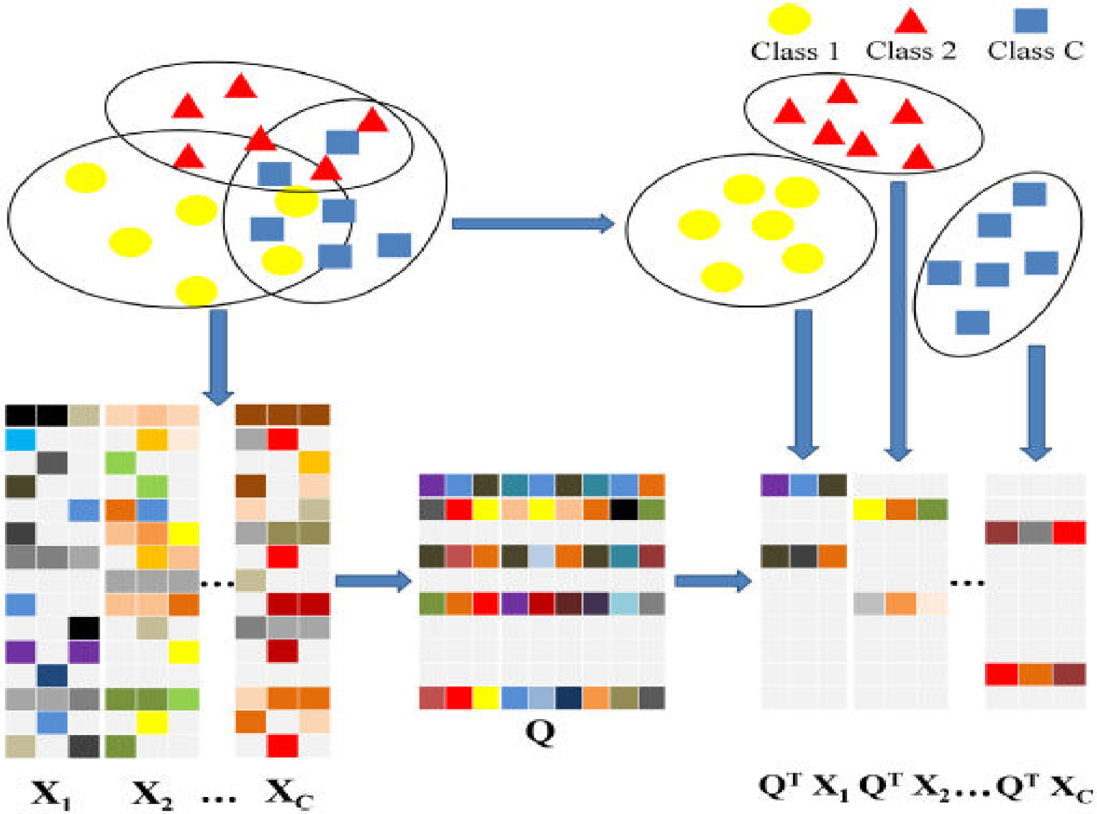
Regularization term for Feature Selection (Reduces over-fitting)

compensate the random and sparse noise.

Provides Inter-class Sparsity

Proposed Method

$$f(\mathbf{Q},\mathbf{E},\mathbf{P},\mathbf{F}) = Tr\left(\mathbf{Q}^T\mathbf{S}\mathbf{Q}\right) + \lambda_1 \|\mathbf{Q}\|_{2,1} + \lambda_2 \|\mathbf{E}\|_1 + \lambda_3 \boxed{\sum_{i=1}^{C} \|\mathbf{F}_i\|_{2,1}}$$



Common sparse structure for the samples in the projected space

$$s.t. \boxed{\mathbf{F} = \mathbf{Q}^T\mathbf{X},} \quad \boxed{\mathbf{X} = \mathbf{P}\mathbf{Q}^T\mathbf{X} + \mathbf{E},} \text{ and } \mathbf{P}^T\mathbf{P} = \mathbf{I}$$

Robust Discriminant Analysis with feature selection and Inter-class Sparsity (RDA_FSIS)

- **Input:** Data Samples: $\mathbf{X}$; Data Labels; Regularization parameters: $\lambda_1, \lambda_2, \lambda_3$.
- **Output:** Transformation matrix $\boldsymbol{Q}$; Reconstruction orthogonal matrix $\boldsymbol{P}$; Error matrix $\boldsymbol{E}$;

- Initialize $\boldsymbol{Q}$, $\boldsymbol{E}$ = 0 ; $\boldsymbol{F} = \boldsymbol{Q}^T \boldsymbol{X}$; $\boldsymbol{P} = \boldsymbol{arg} \min_{\boldsymbol{P}} \boldsymbol{Tr}(\boldsymbol{P}^T \boldsymbol{S} \boldsymbol{P})$; $\boldsymbol{S}_w, \boldsymbol{S}_b$ obtained from LDA.

- Iteratively update $\mathbf{Q}, \mathbf{P}, \mathbf{E}$ and $\mathbf{F}$ (until convergence);

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Compared methods

- K nearest neighbors (KNN)
- Support Vector Machines (SVM)
- Linear Discriminant Analysis (LDA)
- Local Discriminant Embedding (LDE)
- Automatic Subspace Learning via Principal Coefficients Embedding (PCE) (unsupervised)
- Inter-class Sparsity based discriminative least square regression (ICS DLSR) [Wen et al.,2018]
- Robust sparse LDA (RSLDA) [Wen, Fang et al., 2018]

Datasets

Brief Datasets Description					
Dataset	Type	Num Samples	Num of features	Num of classes	Descriptor
Extended Yale B	Face	2414	1024	38	RAW-brightness
COIL 20	Objects	1440	177	20	LBP
LFWA	Face	3408	1024	141	RAW-brightness
MNIST	Digits	60,000	2048	10	Deep Features
PubFig83	Face	13,002	2048	83	HOG,LBP,GABOR

Classification Tables

Table II: Mean classification accuracies (%) of different methods on the Extended Yale B dataset.

No	KNN	SVM	LDA	LDE	ELDE	PCE	SULDA	MPDA	ICS_DLSR	RSLDA	RDA_FSIS
10	69.8	73.85	82.32	79.92	85.85	86.39	84.61	83.67	86.56	86.79	**88.27**
15	75.2	80.02	86.76	83.77	89.30	89.23	88.72	86.82	89.53	89.93	**91.73**
20	80.24	85.79	90.7	88.44	93.07	92.19	91.66	90.38	93.14	93.59	**95.11**
25	82.24	89.03	92.17	90.43	94.09	93.35	92.14	91.79	94.50	94.92	**96.23**

Table IV: Mean classification accuracies (%) of different methods on the LFW-a dataset.

No	KNN	SVM	LDA	LDE	PCE	ICS_DLSR	RSLDA	RDA_FSIS
5	9.90	12.72	20.51	9.98	9.44	22.56	24.70	**28.07**
6	10.57	13.61	25.28	10.49	10.26	25.72	28.42	**30.98**
7	11.06	14.70	28.62	11.24	10.98	29.04	31.50	**33.28**
8	11.35	15.72	32.42	11.71	11.73	31.92	32.48	**35.80**

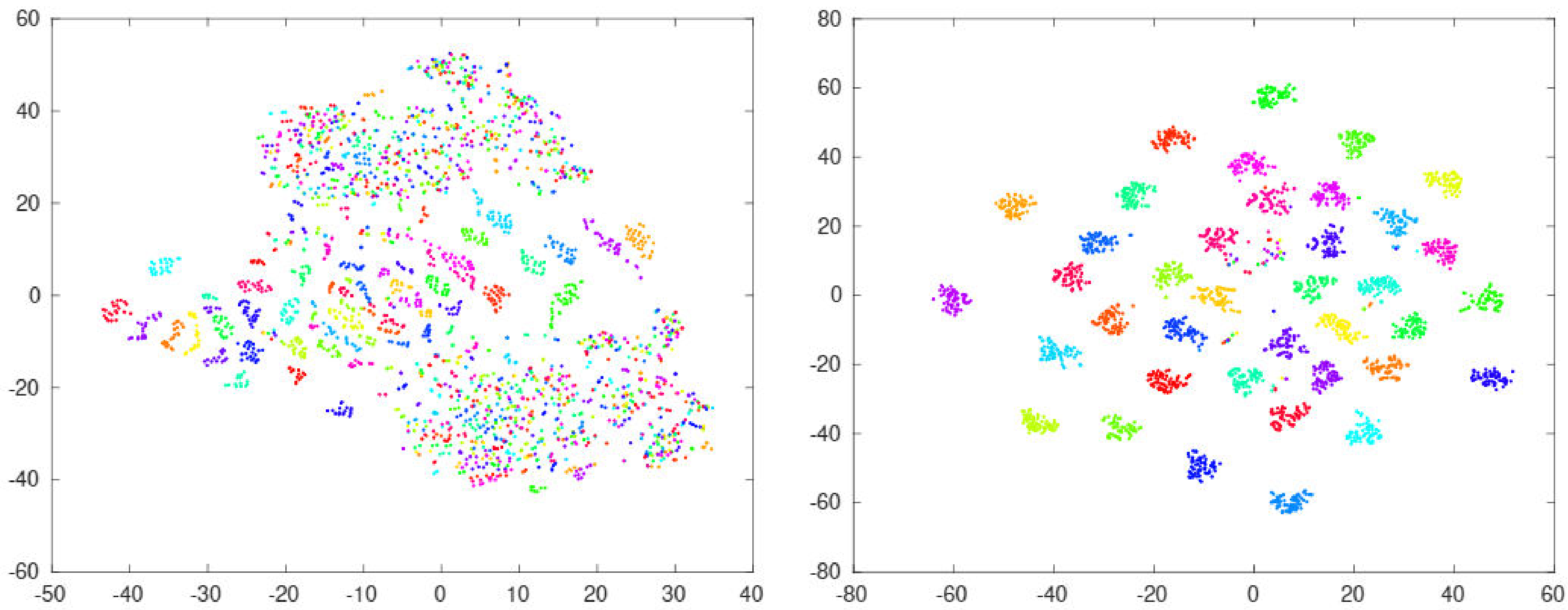
Table V: Mean classification accuracies (%) of different methods on the MNIST dataset.

KNN	SVM	LDA	LDE	PCE	ICS_DLSR	RSLDA	RDA_FSIS
91.75	97.58	85.74	93.22	93.77	98.02	97.95	**98.25**

Table VI: Mean classification accuracies (%) of different methods on the PubFig83 dataset.

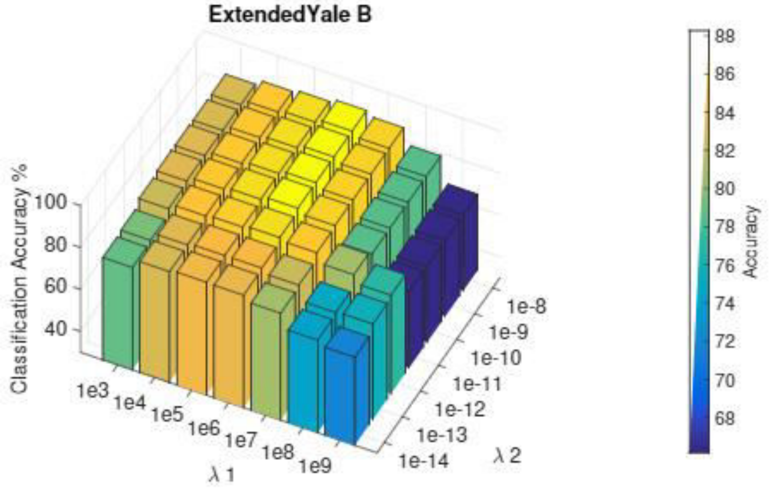
Method	Classification accuracy
KNN	63.35
SVM	82.60
LDA	77.95
LDE	62.89
ELDE	65.88
PCE	50.40
SULDA	81.26
MPDA	67.89
ICS_DLSR	85.19
RSLDA	84.78
DeepLDA	44.35
Alexnet	64.00
Resnet50	**90.40**
RDA_FSIS	84.84

T-SNE Visualization

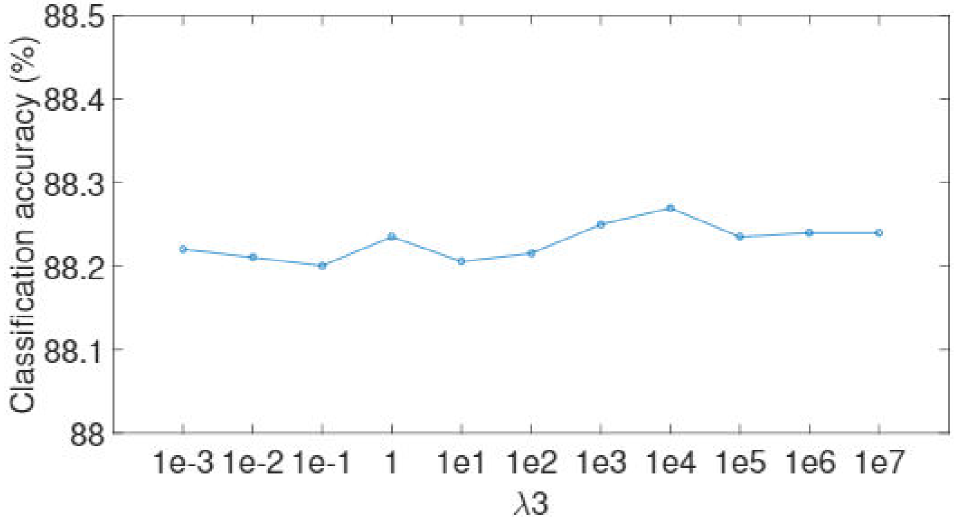


(a) Original features. (b) Projected features.

Parameters Analysis

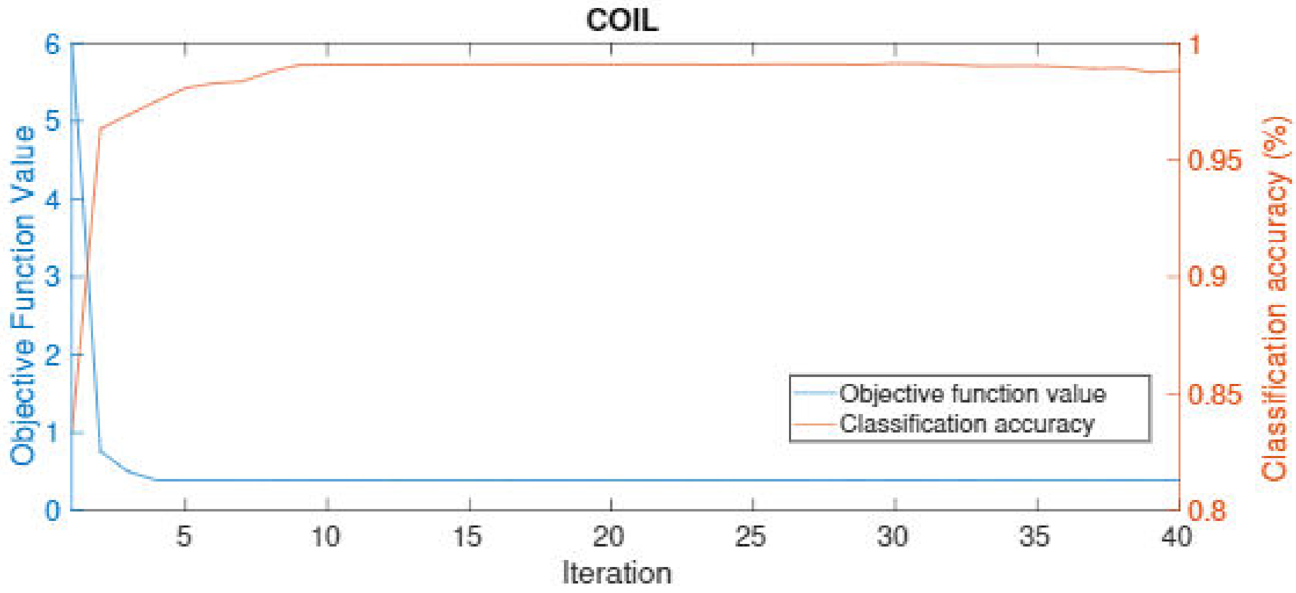


(a)



(b)

Convergence Analysis



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Conclusion

- **Supervised learning with Feature Selection and class sparsity algorithm is proposed**
  - We propose an novel framework that provides an informative and discriminative linear embedding space.
  - Framework that combines Linear discriminant analysis and inter-class sparsity.
  - Implicitly performs feature ranking.
- **The experiments show that the proposed method**
  - is able to outperform competing methods.
  - Robust to noise.
  - Future works will consider transforming this model into a deep model.

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Thank you for your attention

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