

Nonlinear Ranking Loss on **Riemannian Potato** Embedding

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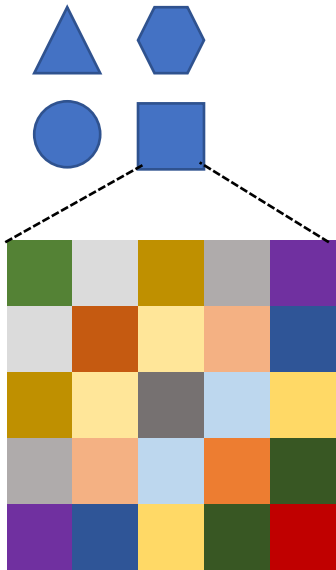
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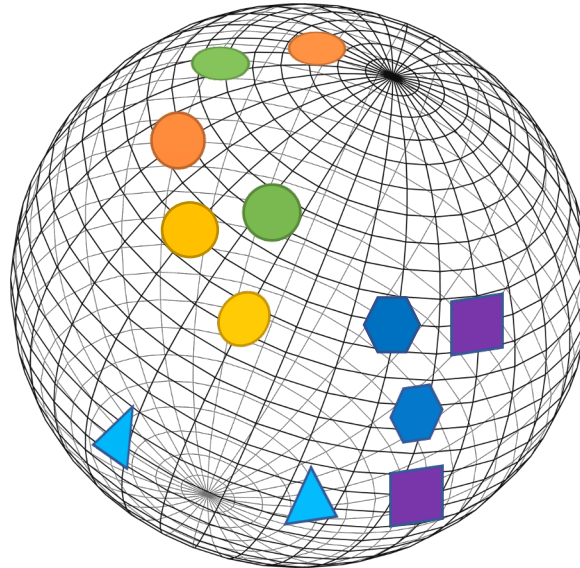
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- Goal : Aims to **pull** positive points closer than the potato-shaped region of acceptability (z-score) and **push** negative points out of the boundary.
- Contribution
 - We propose a new ranking loss to learn discriminative embeddings on **non-Euclidean** spaces exploiting **the structure of non-linear embedding spaces**.
 - We achieve new state-of-the-art performance on three popular benchmarks, **reducing** the intra-class distances and **enlarging** the inter-class distances for the learned features.

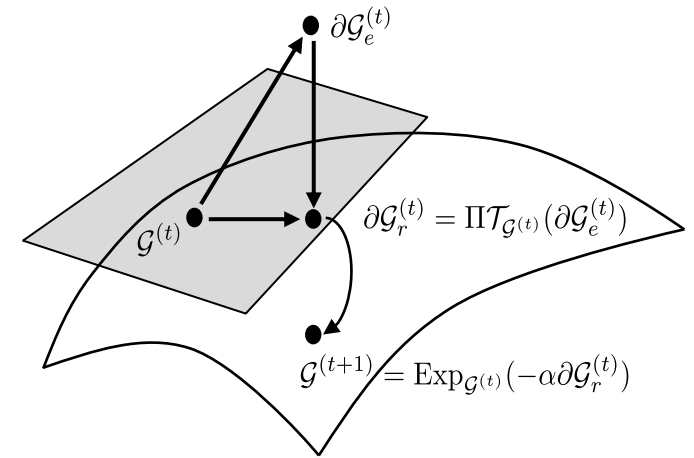
Covariance matrix
representation

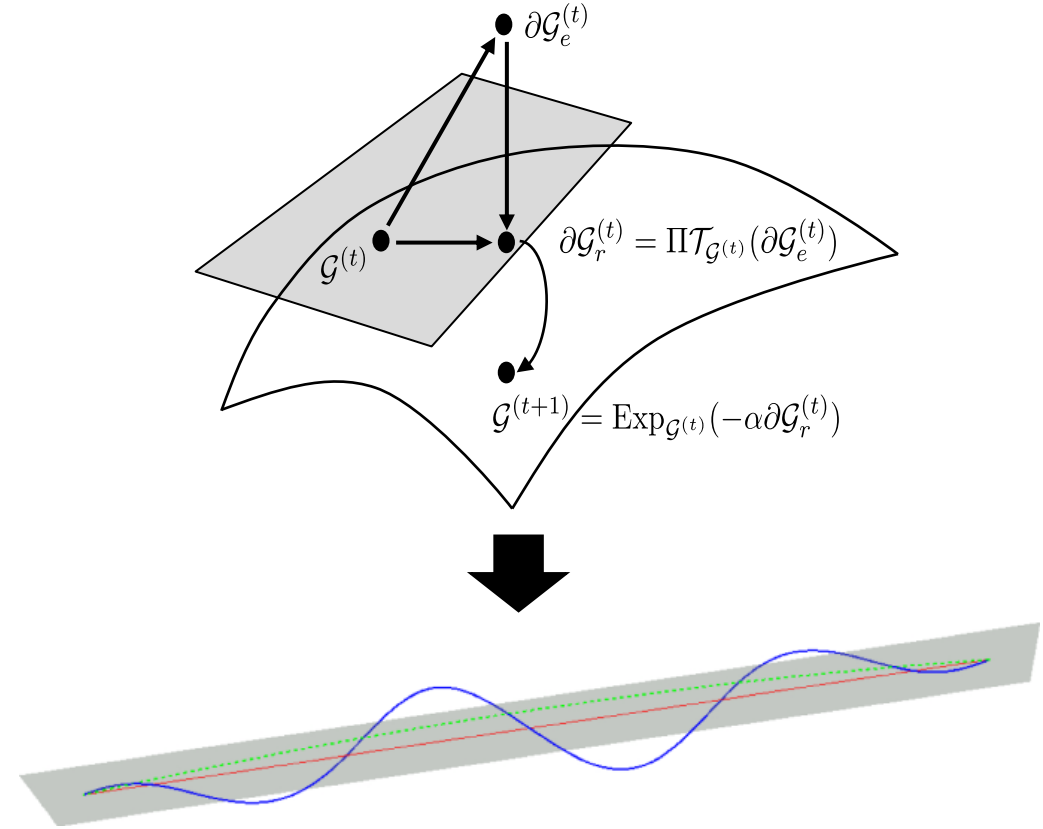
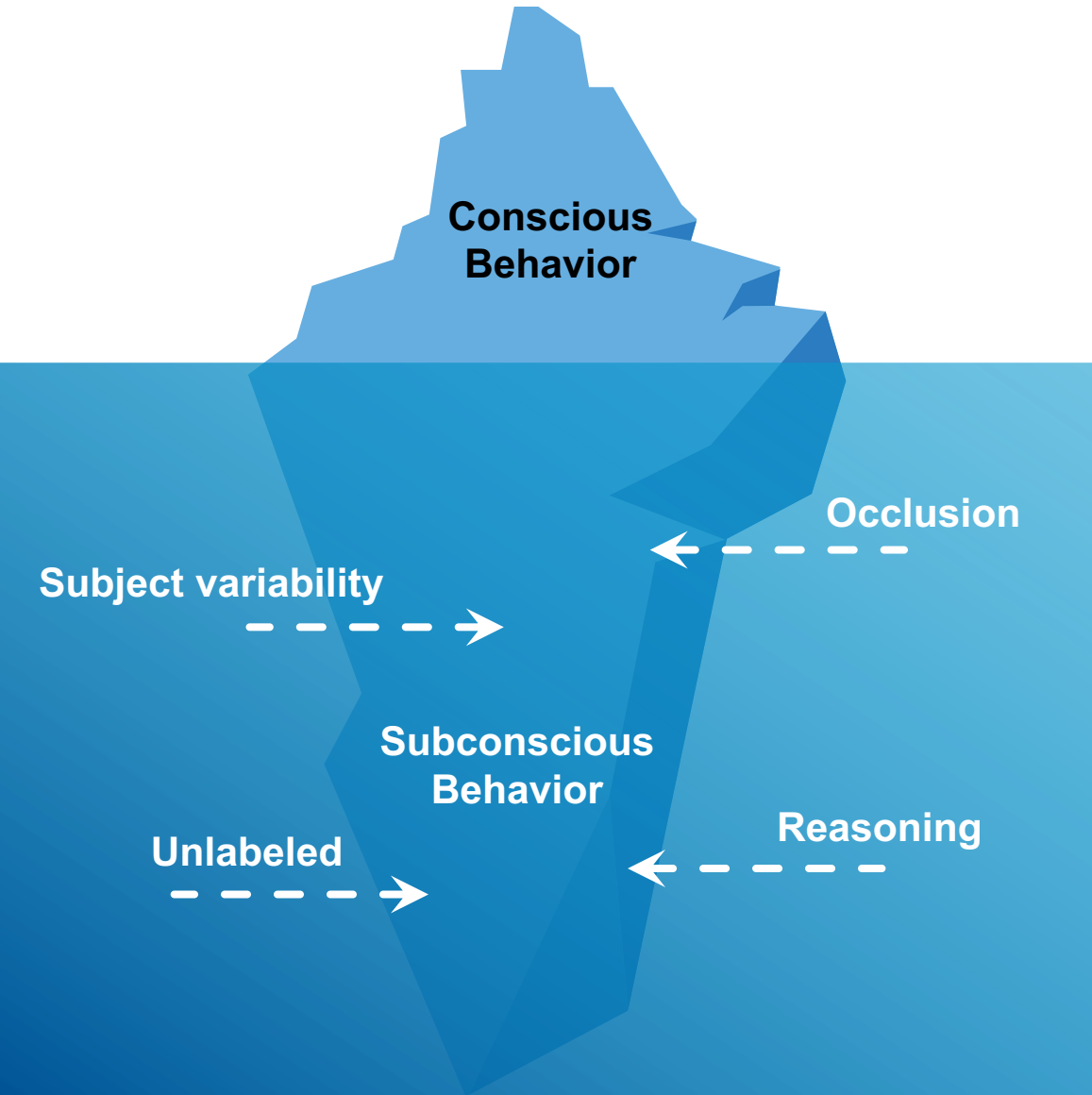


Riemannian Geometry
over a SPD manifold



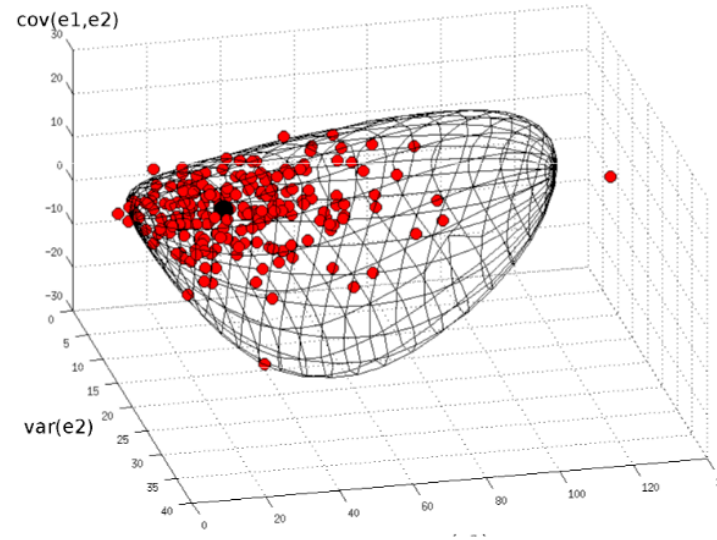
Tangent
Approximation





$$L(X, y) = \frac{1}{\mathcal{T}} \sum_{(i,j,k) \in \mathcal{T}} [d_{(i,j)}^2 - d_{(i,k)}^2 + \alpha]_+$$

- Motivation
 - Riemannian Potato : Provides a measure of **dispersion** using the distribution of distances between covariance matrices and a reference matrix and **rejects** epochs whose covariance matrices lie out of a region of **acceptability** defined by a **z-score** threshold.



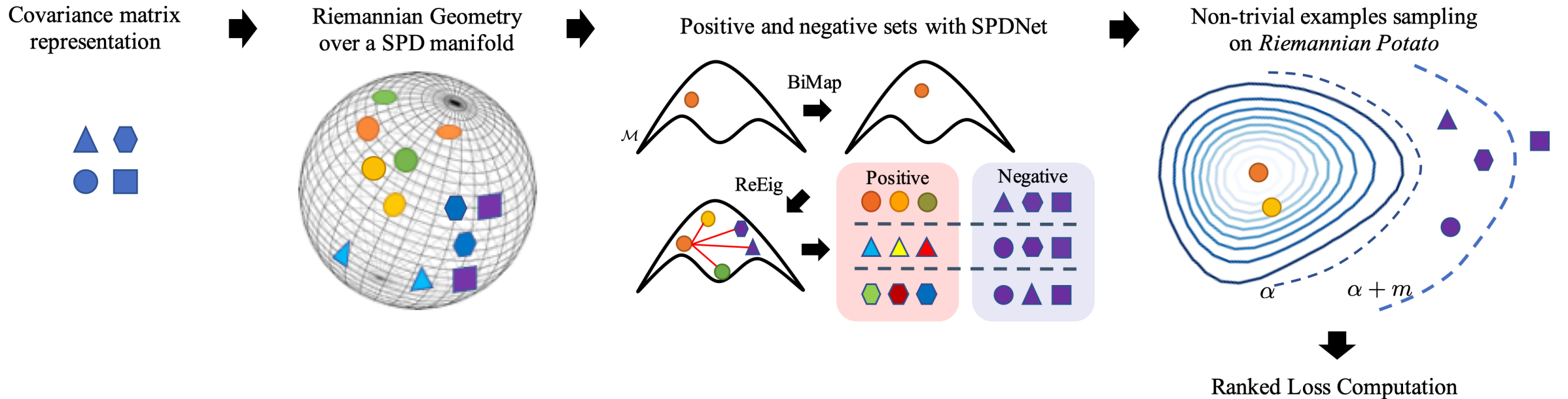
$$z_t = \frac{\log(d_t/\mu_t)}{\log(\sigma_t)} \quad d_t = \delta_R(\Sigma_t, \bar{\Sigma}_{t-1})$$

$$\bar{\Sigma} = \arg \min_{\Sigma \in \mathcal{M}} \sum_{i=1}^{N_I} \delta_R^2(\Sigma_i, \Sigma)$$

$$\mu_t = \exp\left(\frac{1}{t} \sum_{i=1}^t \log(d_i)\right),$$

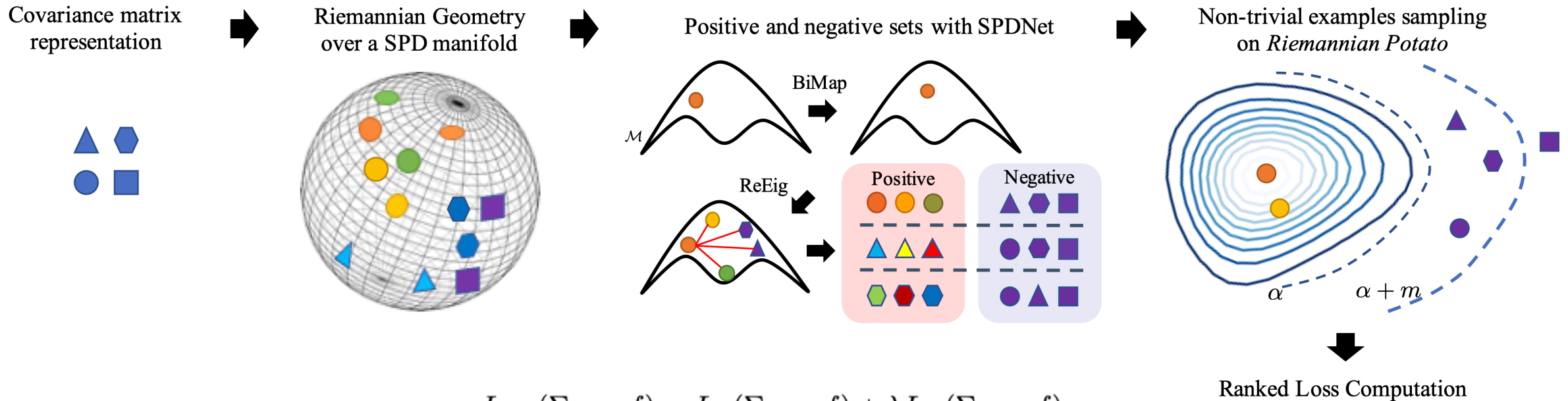
$$\sigma_t = \exp\left(\sqrt{\frac{1}{t} \sum_{i=1}^t (\log(d_i/\mu_i))^2}\right)$$

- Riemannian Potato-based Ranking Loss (RPL)
 - A **rank**-based metric learning method for learning discriminative embeddings



$$\begin{aligned}
 \mathcal{P}_i^c &= \{\forall \Sigma_j | j \neq i \wedge c = y_i = y_j\} & \hat{\mathcal{P}}_i^c &= \{\forall \Sigma_j | j \neq i \wedge c = y_i = y_j, z_j^c > z_{th}\} & L(\Sigma_i, \Sigma_j, y_i; f) &= (1 - y_{ij})[z_{th} - z_j^c] \\
 \mathcal{N}_i^c &= \{\forall \Sigma_j | c = y_i, y_i \neq y_j\} & \hat{\mathcal{N}}_i^c &= \{\forall \Sigma_j | c = y_i, y_i \neq y_j, z_j^c < z_{th} + m\} & & + y_{ij}[z_j^c - (z_{th} + m)]_+,
 \end{aligned}$$

- Riemannian Potato-based Ranking Loss (RPL)
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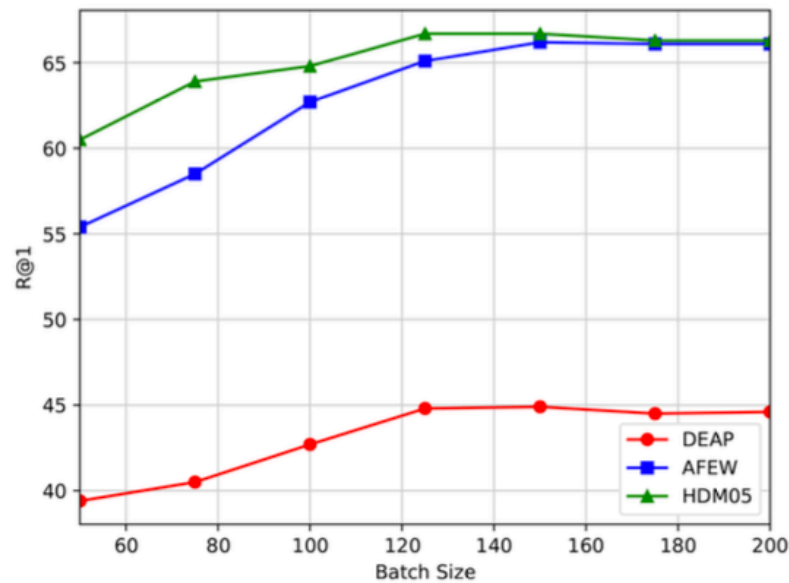
$$L_{RP}(\Sigma_i, y_i; f) = L_P(\Sigma_i, y_i; f) + \lambda L_N(\Sigma_i, y_i; f).$$

$$L_P(\Sigma_i, y_i; f) = \frac{1}{|\hat{\mathcal{P}}_i^c|} \sum_{\Sigma_j \in \hat{\mathcal{P}}_i^c} L(\Sigma_i, \Sigma_j, y_i; f) \quad L_N(\Sigma_i, y_i; f) = \frac{1}{|\hat{\mathcal{N}}_i^c|} \sum_{\Sigma_j \in \hat{\mathcal{N}}_i^c} L(\Sigma_i, \Sigma_j, y_i; f)$$

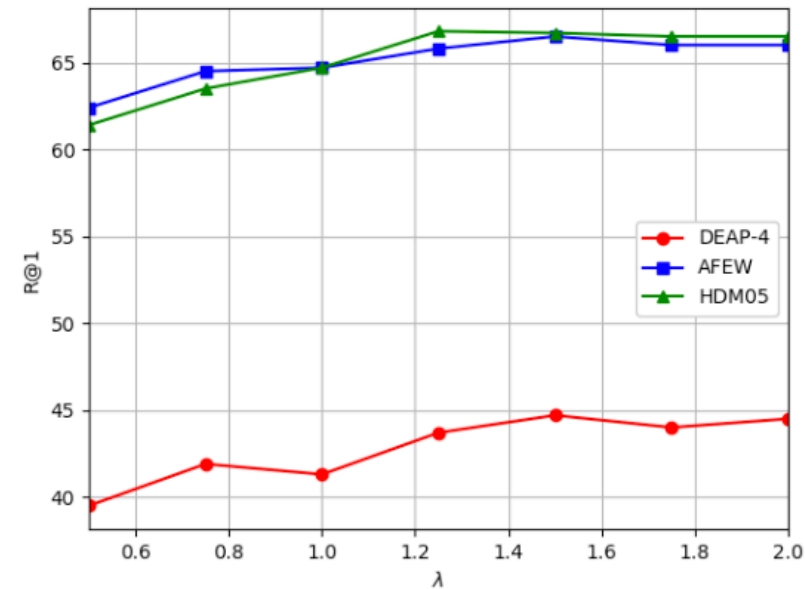
- Public Datasets
 - Emotion Recognition (DEAP, AFEW), Skeleton-based Human Action Recognition (HDM05)

Method	DEAP-4				AFEW				HDM05			
	F1	R@1	R@3	NMI	F1	R@1	R@3	NMI	F1	R@1	R@3	NMI
DSML-Triplet	38.7	35.5	37.8	36.1	59.3	61.3	64.2	61.9	53	57.3	58.5	52.4
Triplet-Random	33.5	31.4	32.5	28.7	55.8	55.1	55.4	57.4	48.3	44.5	47.5	45.6
Triplet-Semihard	35.5	30.1	31.4	27.3	57.4	54.4	60.5	58	51.3	50.4	55.3	47.5
Lifted Struct	35.3	35.2	35.8	33.4	62.5	65.4	68.4	69.4	55.5	59.3	59.2	53.4
N-pair-mc	41.5	38.4	39.5	34.8	66.4	63.5	66.4	65.1	59.8	60	61	59.4
Proxy NCA	39.8	41.3	41.4	38.1	66.5	64.2	66	66.3	59	63.3	64.5	62
NRA	42.2	44.4	46.2	37.2	67.2	66.5	68.6	66.8	59.2	64.3	65.2	64.1
RPL	43.3	44.7	46.2	37.5	67.4	66.5	69.4	66.4	59.4	66.7	68.8	65.4

- The effect of RPL
 - Batch size, Parameter λ



Recall@1 results of different batch size on the three datasets (DEAP, AFEW, and HDM05).



Recall@1 results of different λ on the three datasets (DEAP, AFEW, and HDM05).

- A **rank**-based metric learning method for learning discriminative embeddings
 - Given a query covariance matrix, our RPL **splits** its positive and negative sets and forces a margin between them on a SPD manifold.
 - Non-trivial samples mining and negative examples weighting are exploited to make better use of all **informative** data points.
 - The proposed method achieves state-of-the-art performance, **reducing** the intra-class distances and **enlarging** the inter-class distances for learned features.
- Our next work will study the **non-stationary** nature of brain activity as revealed by EEG
 - subject to noises from various artifacts, low signal-to-noise ratio (SNR) of sensors, and inter- and intra- subject variability.

Thank you