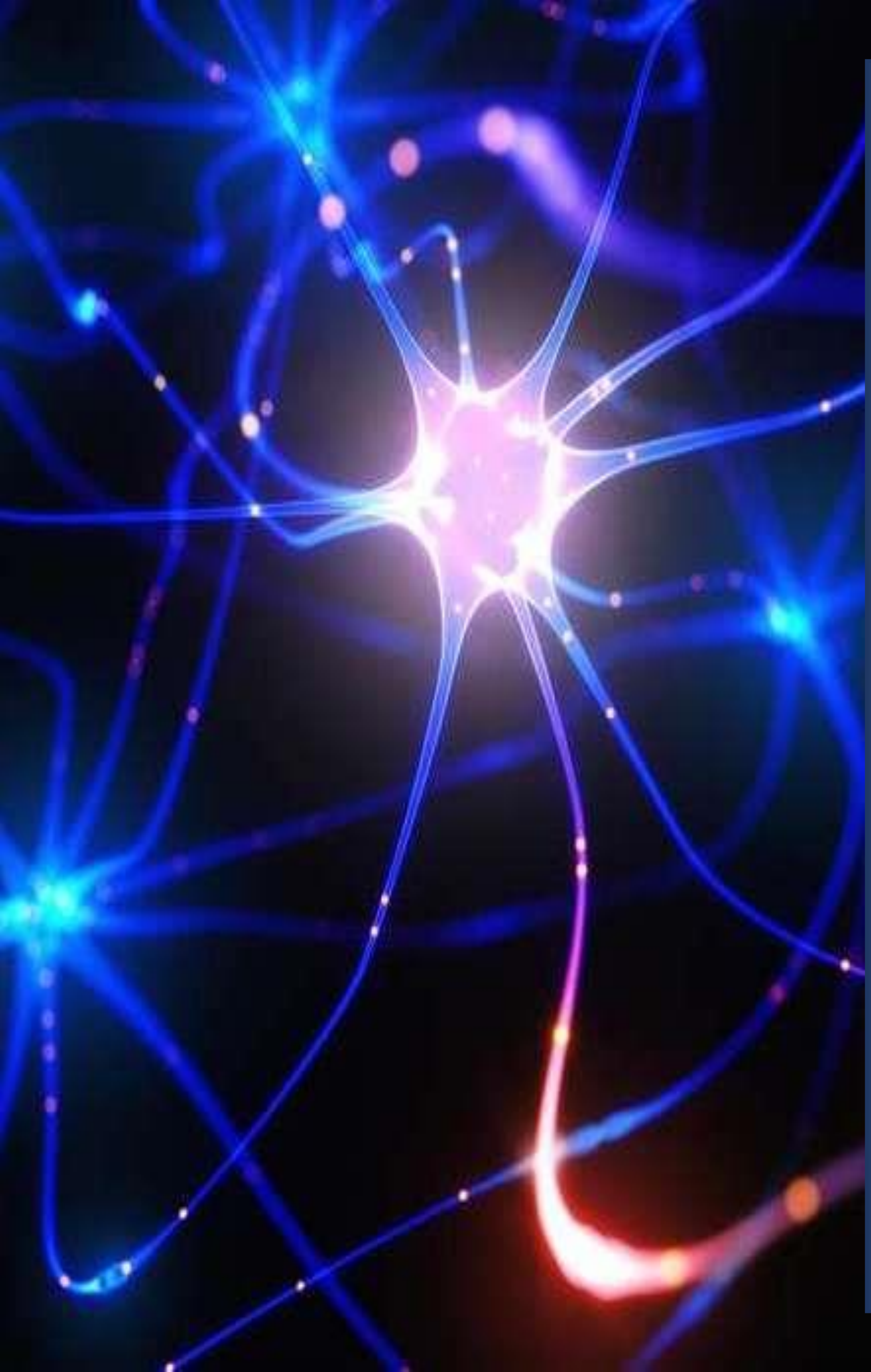

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Estimating Brain Networks by Kulback-Leibler Divergence

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Major Goal

- Represent brain activities by networks.
  - ✓ Estimate the activations between anatomic regions using fMRI data during a complex problem-solving.
- Analyze the brain networks activated in two main phases of the complex problem-solving process:
  - ✓ Planning
  - ✓ Execution

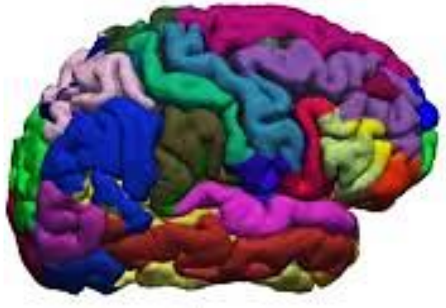
  for
  - ✓ successful
  - ✓ unsuccessful

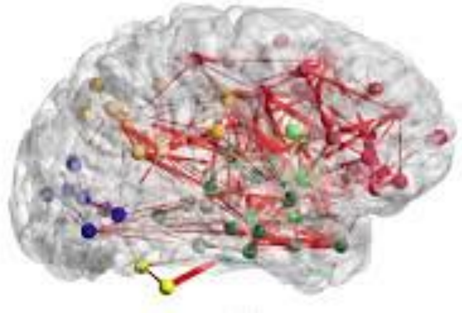
  problem solving sessions.

Method

- A novel method to estimate static and dynamic brain networks using Kulback-Leibler Divergence (Relative Entropy)
- The validity of the estimated brain networks are investigated by analyzing the planning and execution phases of problem-solving process.

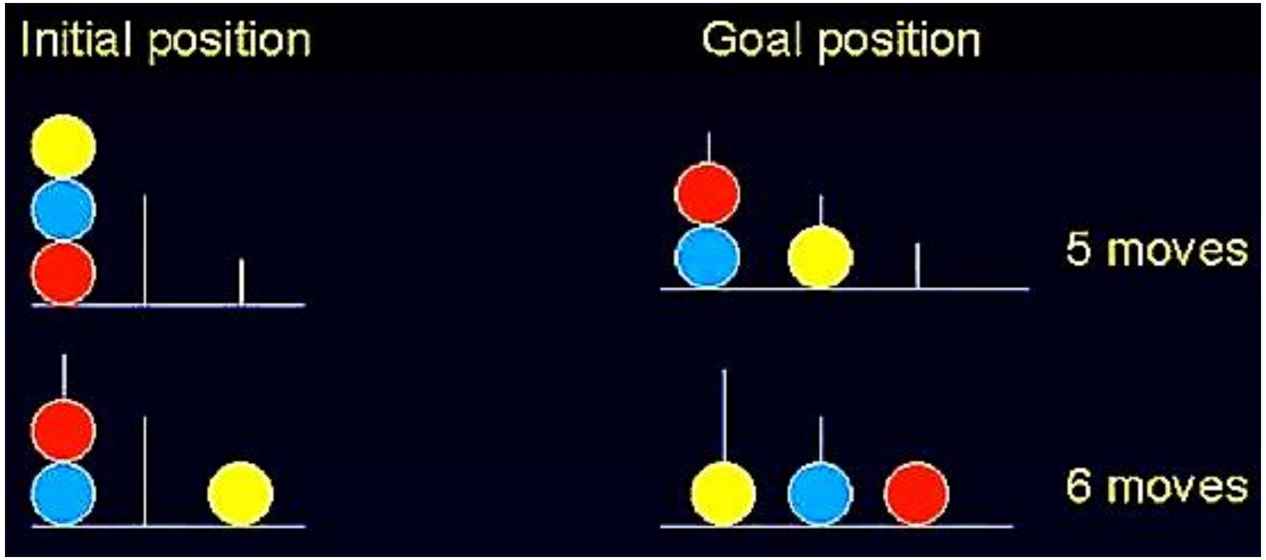
✓ The suggested computational network model is tested by a classification schema using Support Vector Machines.





Tower Of London (TOL) Experiment

- fMRI data was recorded during the computerized **TOL game**.
- The **start** and **goal states** were presented.
- It requires a **five** or **six moves** to reach the goal.
- Subjects were directed to generate a **solution plan** prior to making their first move.



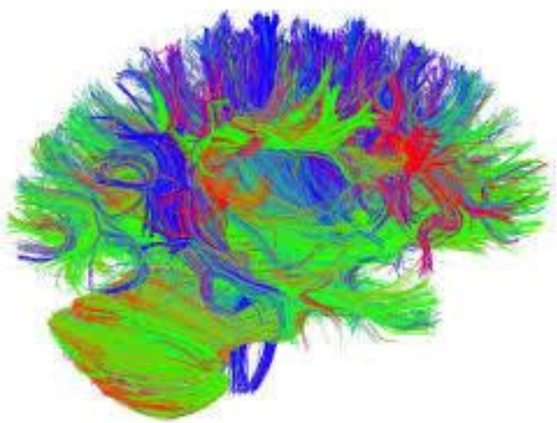
Kullback–Leibler Divergence

- Is a fundamental equation of information theory that quantifies the proximity of two probability distributions.
- Measures dissimilarity / distance between distributions.
- The Kullback–Leibler (K-L) divergence between two probability distributions P and Q is defined to be:

$$D(P\|Q) = \sum_{x \in \mathcal{X}} P(x) \log \frac{P(x)}{Q(x)}$$

Major Assumption

- Our major assumption is, the degree of co-activation between two anatomic regions can be measured by Kullback-Leibler divergence.
- Therefore, the measure of K-L divergence between the anatomic regions can be used as the arc weights of the brain network formed among the regions.
- Based upon this assumption we estimate two types of brain network: **static** and **dynamic** brain networks.



Dynamic Brain Networks

- Dynamic brain networks are estimated from the probability distribution of voxel intensity values for each anatomic region, at each time instant.

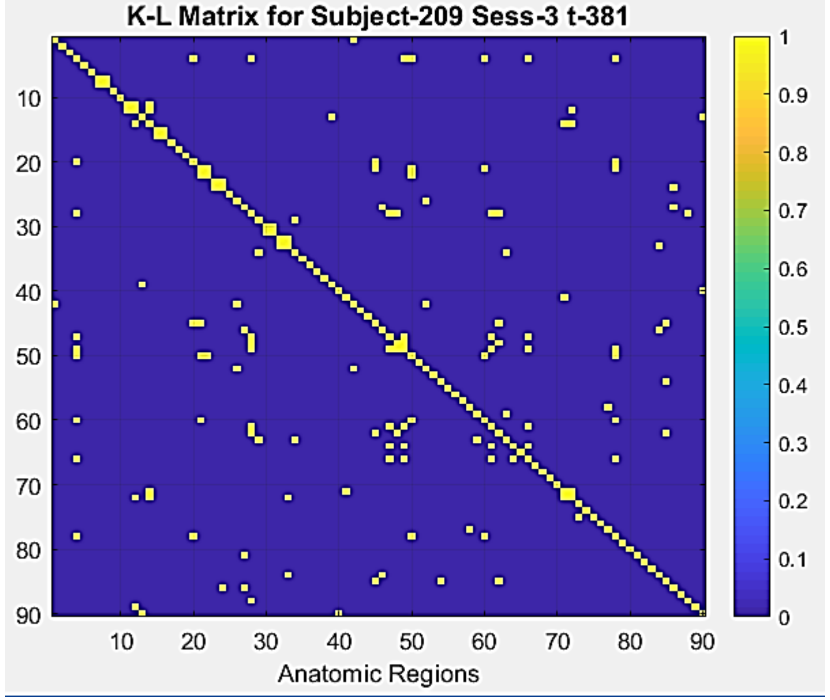
$$P_r(v(t)) = \frac{1}{n_r h_r} \sum_{i=1}^{n_r} K\left(\frac{v(t) - v_i(t)}{h_r}\right)$$

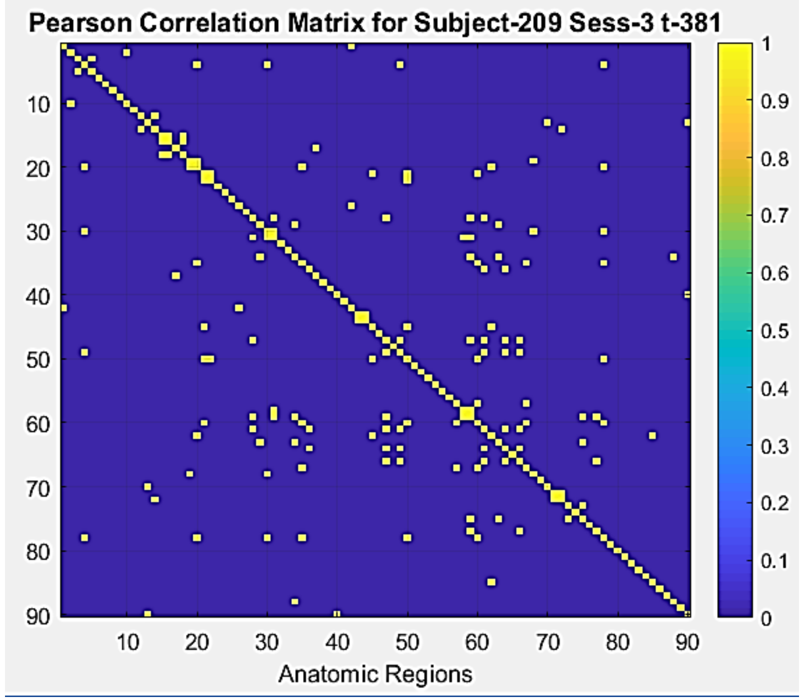
- K-L divergence between the anatomic regions are then estimated for the region pairs.

$$D_{\mathrm{KL}}(P_k(v(t)) \parallel P_l(v(t))) = -\sum_{v(t)\in\mathcal{X}} P_k(v(t)) \log\left(\frac{P_k(v(t))}{P_l(v(t))}\right)$$

Comparison of K-L and Pearson Matrices

- The K-L distance matrices and Pearson correlation matrices are estimated for each time-instant.
  - ✓ Although the K-L distance and Pearson r values are different metrices, the two matrices display similar pattern.





Static Brain Networks

- The static brain networks are estimated for
  - ✓ planning and execution tasks,
  - ✓ successful and unsuccessful problem-solving sessions.
- Each anatomic region is represented by a time series by averaging all the voxel time series resides in that region.
- A probability distribution function is estimated for each region and cognitive state.

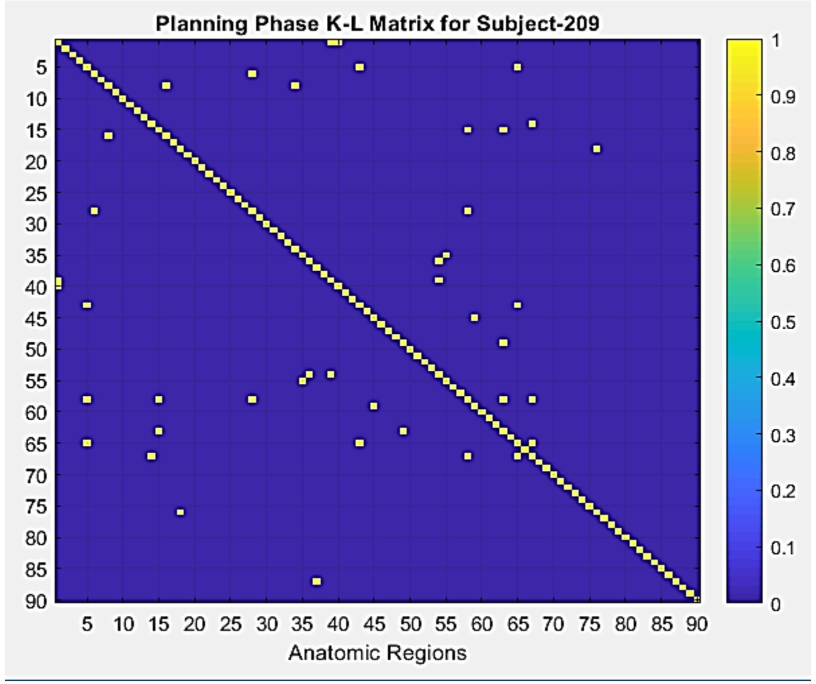
$$X_r(t) = \frac{1}{n_r} \sum_{\forall v_i \in r} v_i(t)\,; \qquad P_{p,r}(x) = \frac{1}{h_r n_p} \sum_{t=1}^{n_p} K\left(\frac{x - X_r(t)}{h_r}\right)$$

- K-L divergence between the anatomic regions are estimated for the region pairs.

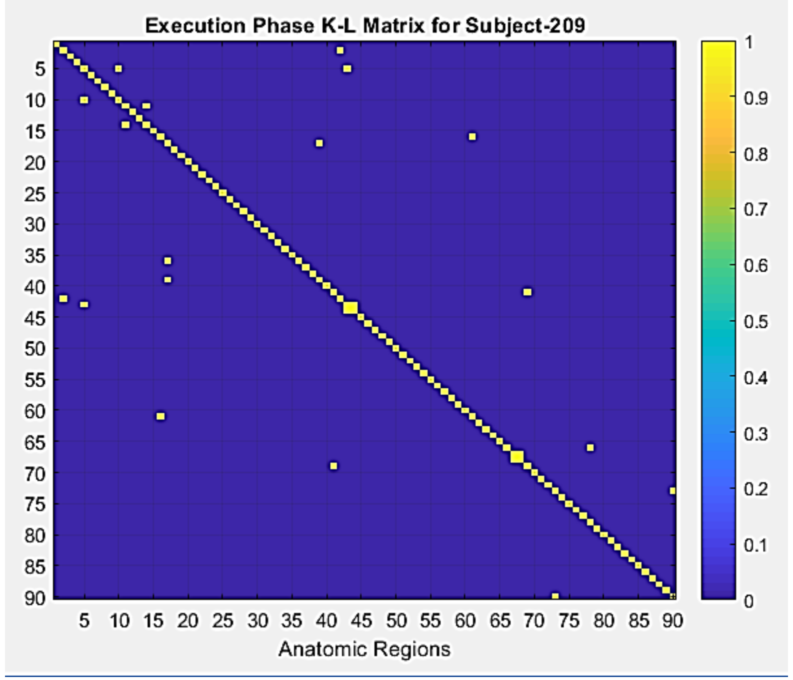
K-L Matrices for Planning and Execution Tasks

The shortest K-L distances between anatomic regions for planning versus execution tasks for a subject.

✓ Number of min K-L distances between regions for the planning, is higher than the number of min K-L distances for the execution.

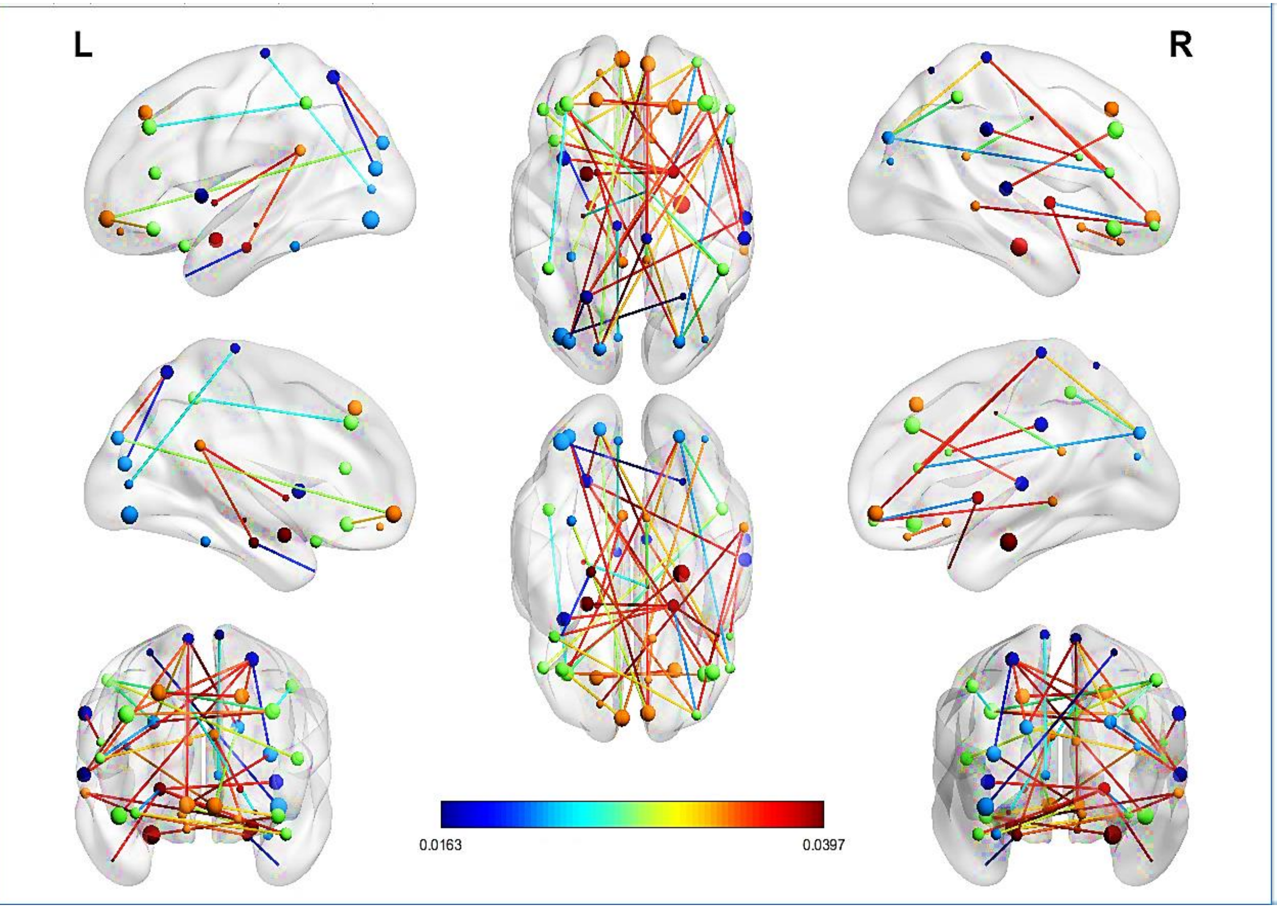


Planning

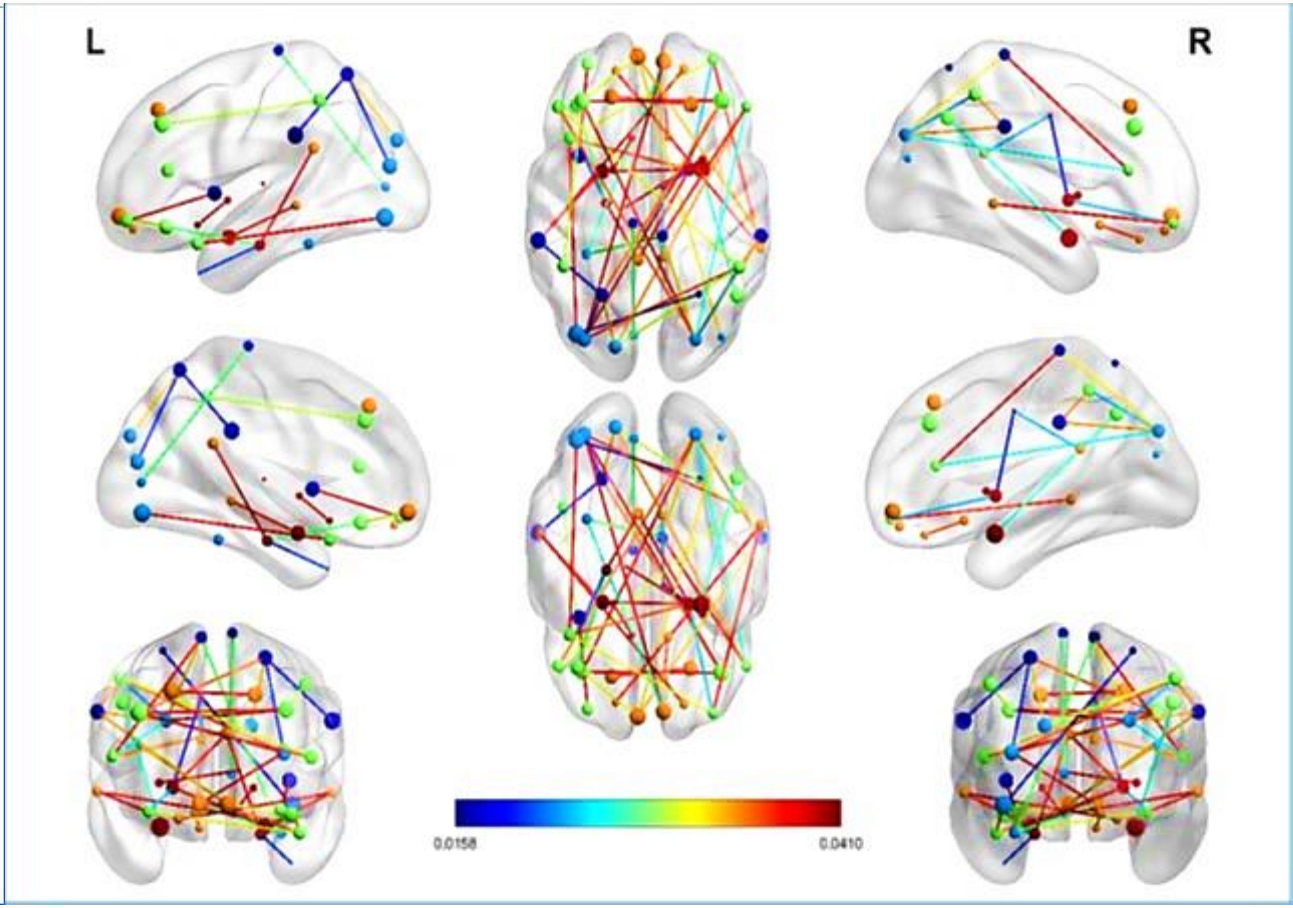


Execution

Important Connections for Successful Sessions



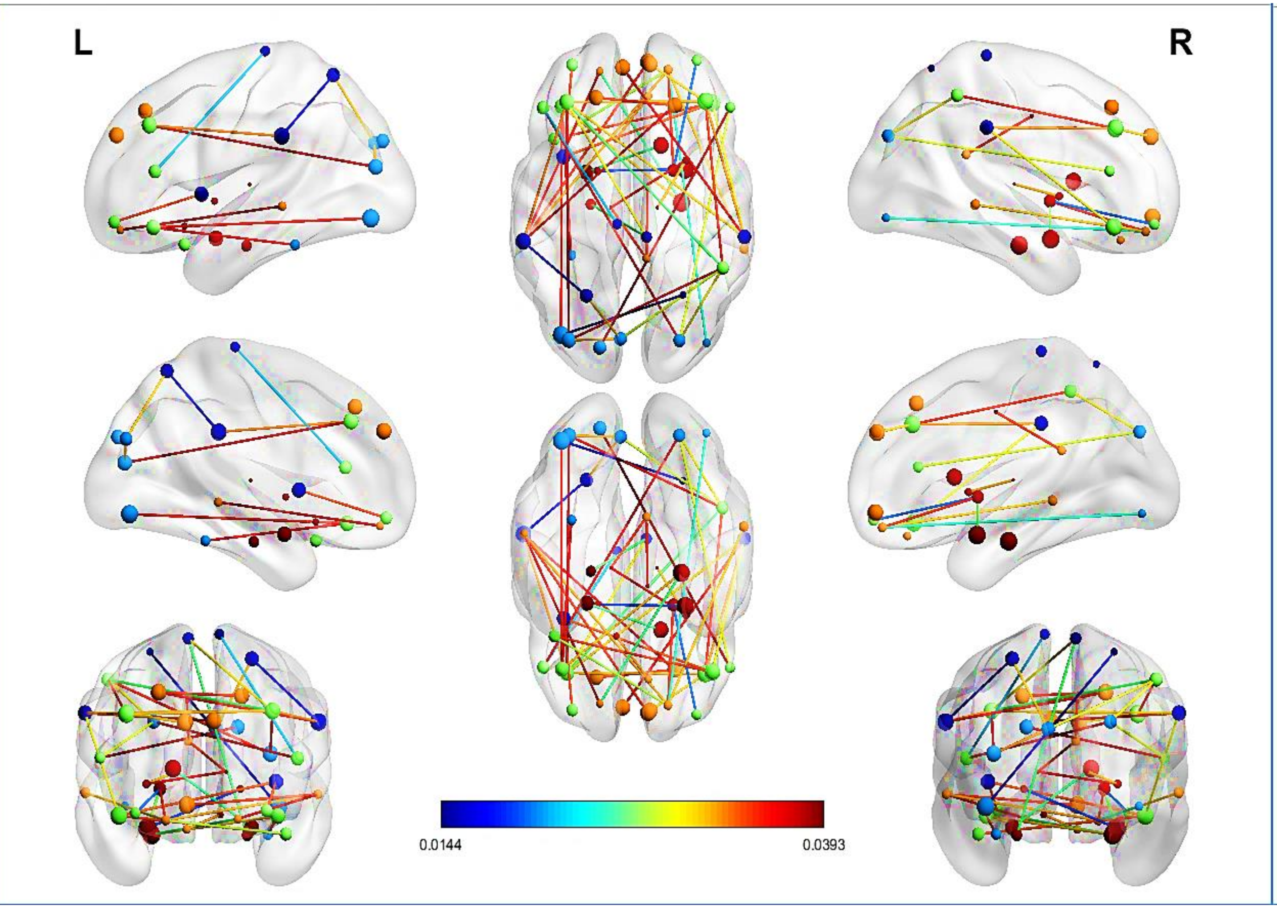
Planning Phase

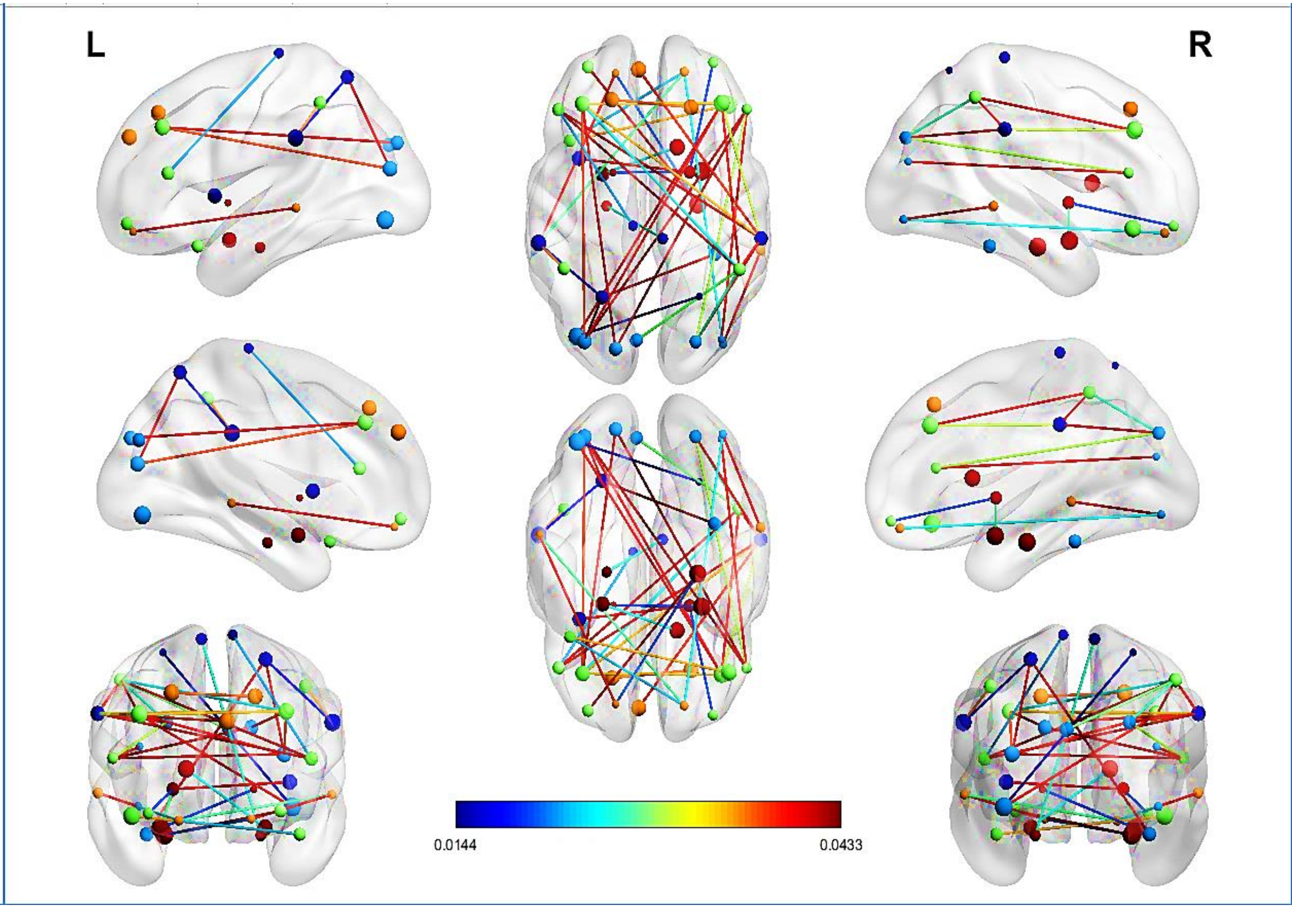


Execution Phase

The size of nodes was set according to the node degree (the number of connections a node has with other nodes). Blue color represent shortest distances.

Important Connections for Unsuccessful Sessions





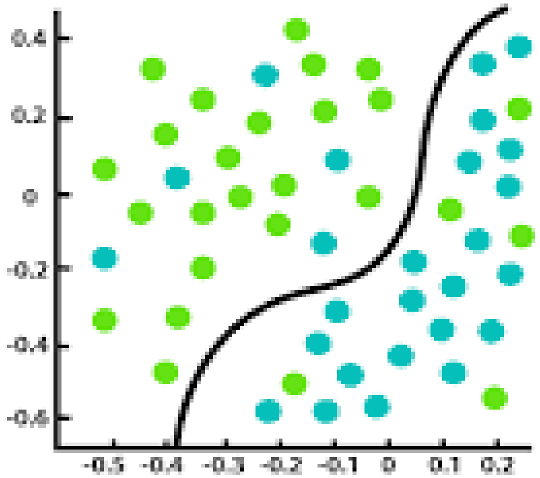
Planning Phase

Execution Phase

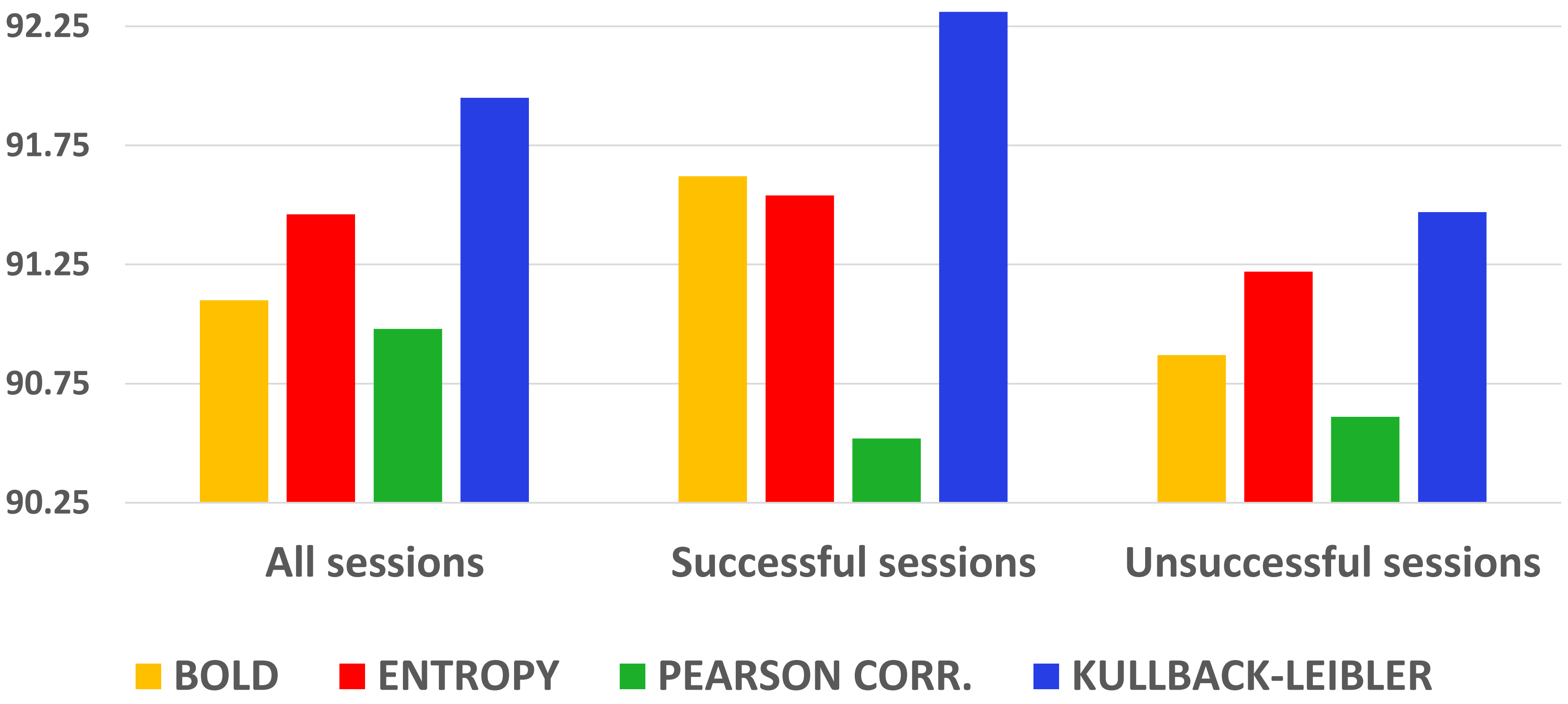
The size of nodes was set according to the node degree (the number of connections a node has with other nodes). Blue color represent shortest distances.

Classification Of Planning and Execution

- We test the validity of the K-L brain networks by training a classifier with the arc-weights.
- In order to compare the different methods, we measure the classification performances for the planning and execution tasks using
  - BOLD values,
  - 1st order Entropy values,
  - Pearson correlation coefficients,
  - K-L divergence values.

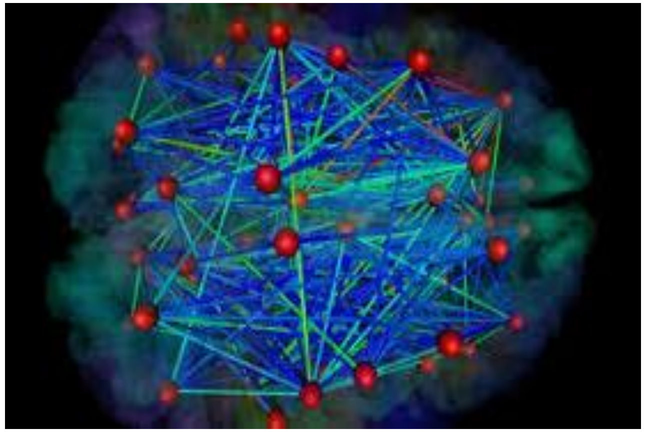


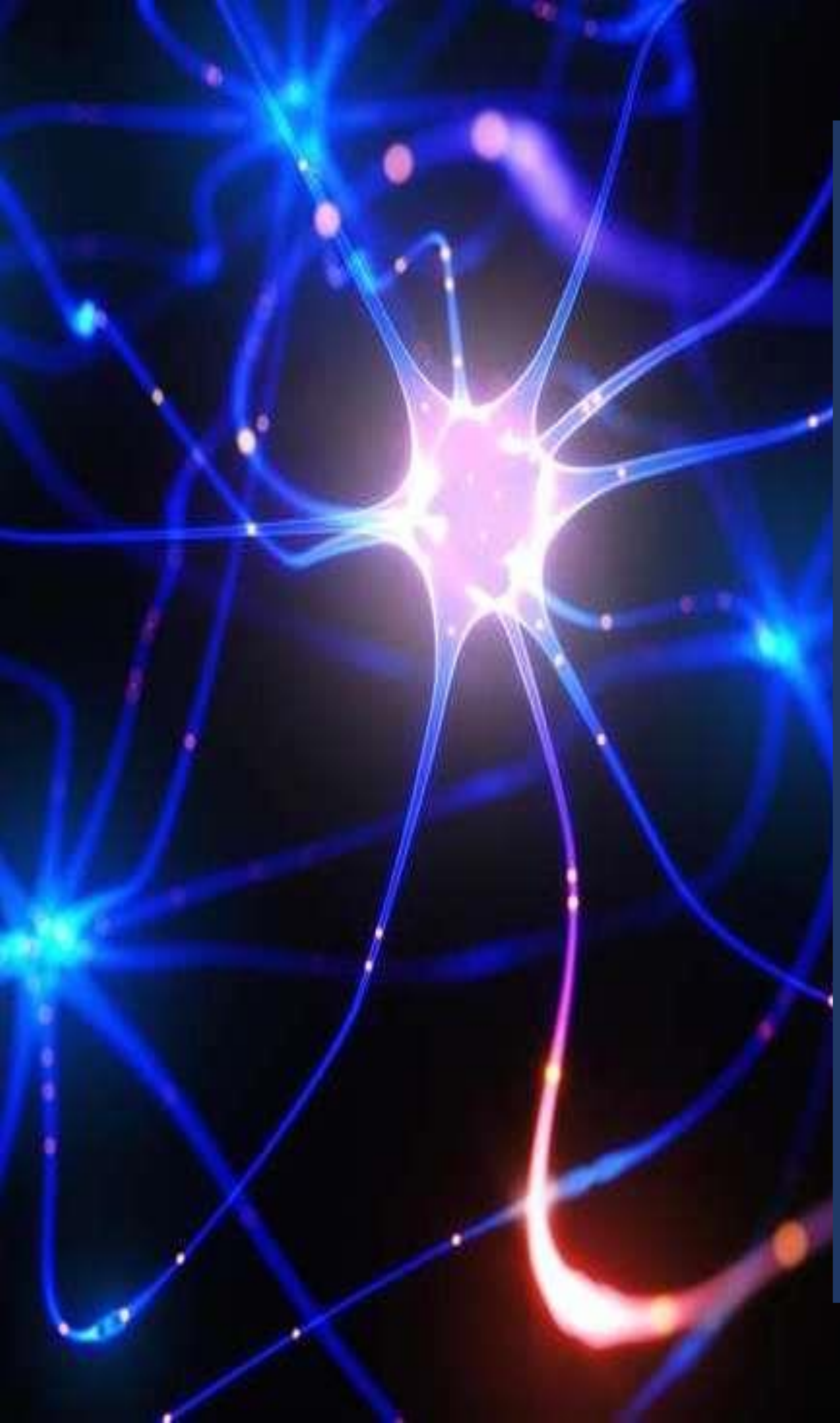
SVM Classification Accuracy For All Subjects



Conclusion

- Kullback-Leibler divergence provides an information theoretic tool to estimate brain networks.
- Results show strong promise in using the Kullback-Leibler networks as a measure for characterizing the brain states for a cognitive task.





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Thank You!