Boundaries of Single-Class Regions in the Input Space of Piece-Wise Linear Neural Networks

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Artificial Neuron

$$L_i^{l+1} = ReLU\left(\sum_j^{n_l} w_{ji} \cdot L_j^l + b_i^{l+1}\right)$$

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$$if \ \sum w_{ij} \cdot L_j^l + b_i^{l+1} \geq 0$$

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2. Hyperplanes

For each ReLU activation,

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can be expressed
in term of input pixels

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$$\sum \alpha_j \cdot I_j + c \geq 0$$

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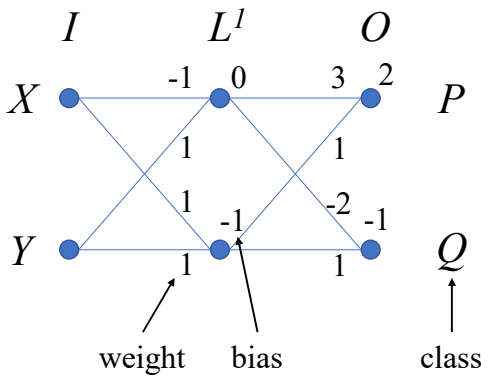
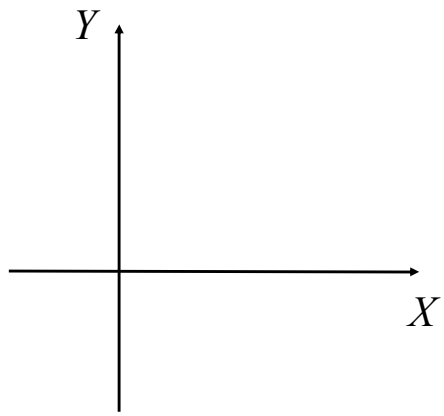
Hyperplane defined by $\sum \alpha_j \cdot I_j + c = 0$ divides input space into two regions.

Toy Example





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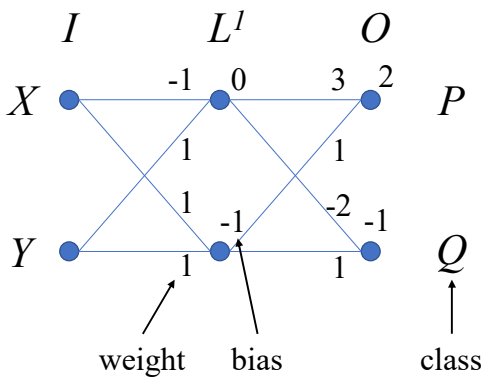
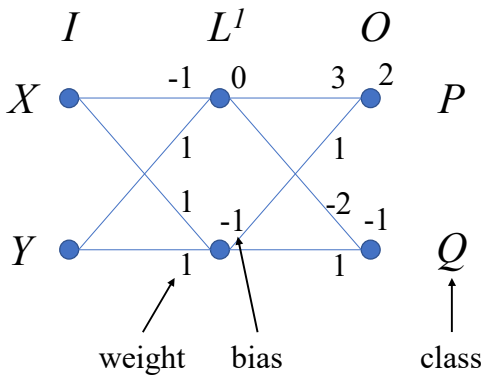
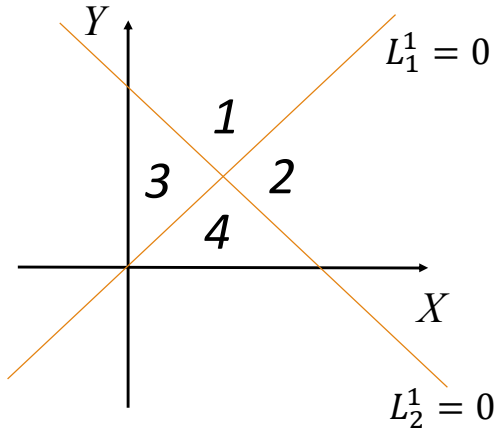


$$L_1^1 = -X + Y + 0$$
$$L_2^1 = X + Y - 1$$

1. $L_1^1 \geq 0, L_2^1 \geq 0$
2. $L_1^1 = 0, L_2^1 \geq 0$
3. $L_1^1 \geq 0,\ L_2^1 = 0$
4. $L_1^1 = 0,\ L_2^1 = 0$

Toy Example





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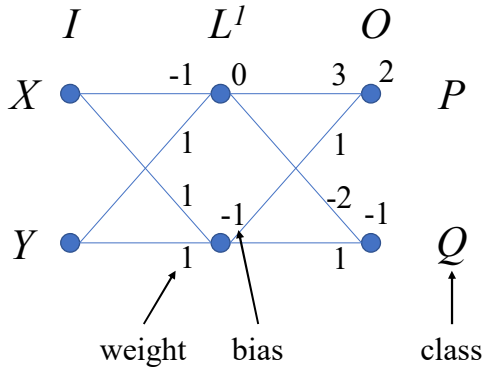
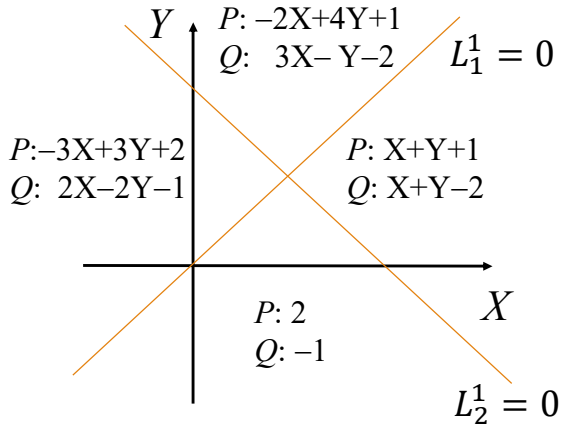




Partition (linear regions)

Partitions are

- bounded

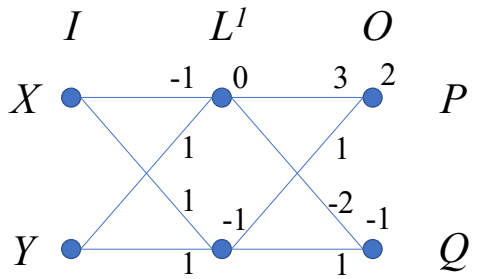
  the input space is bounded

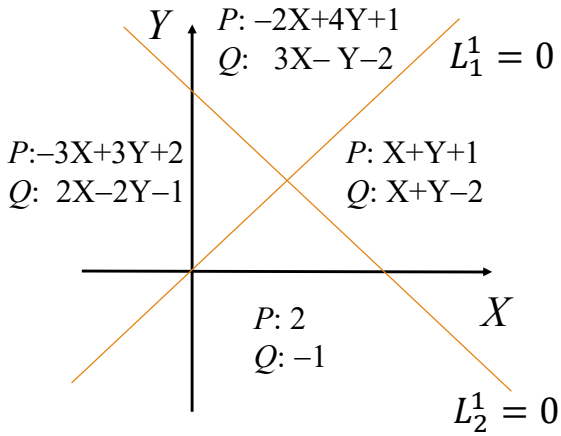
- convex regions

  the hyperplane are defined by linear inequality

Partitions are

- shared by all classes





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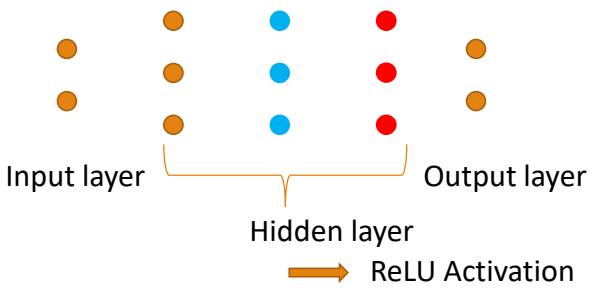
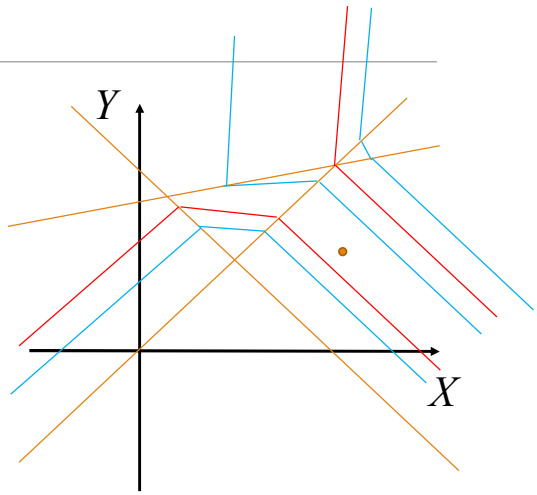


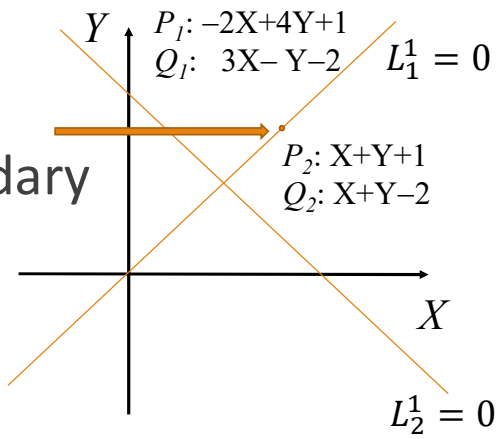


Outputs are

- continuous at the partition boundary

EX) $P_1 = P_2$

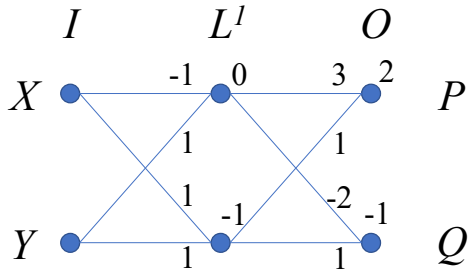
$Q_1 = Q_2$



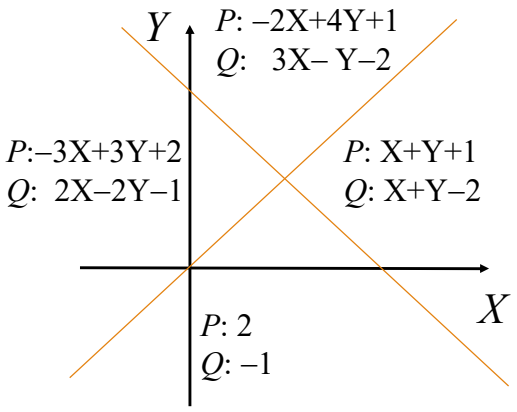
Class Boundary (Decision Boundary)

Class Boundary

- $P > Q \rightarrow P$
- $P - Q > 0$
- $P - Q = f(I) + c$



Class Boundary



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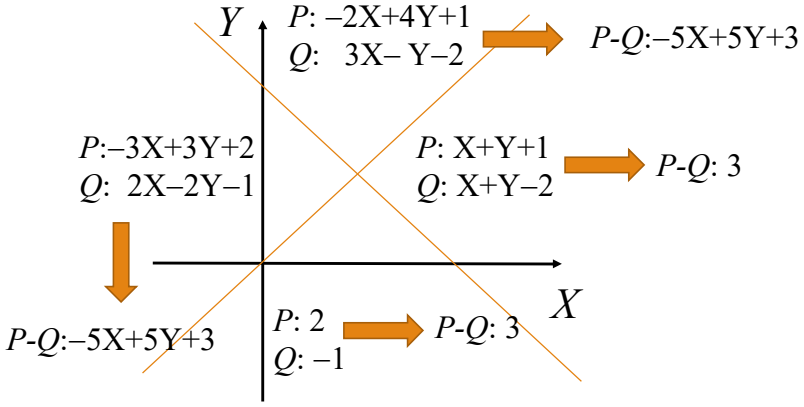

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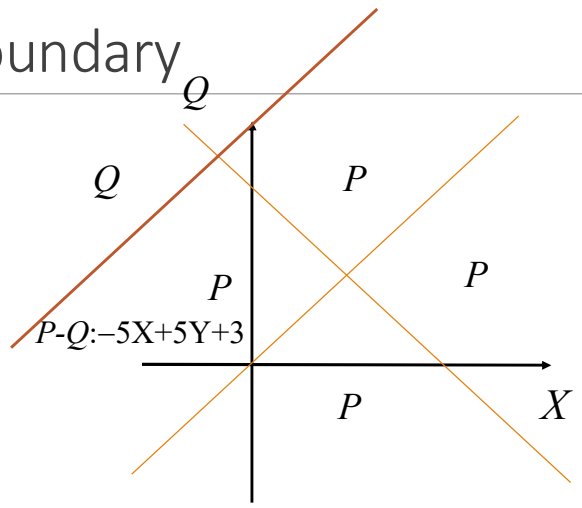

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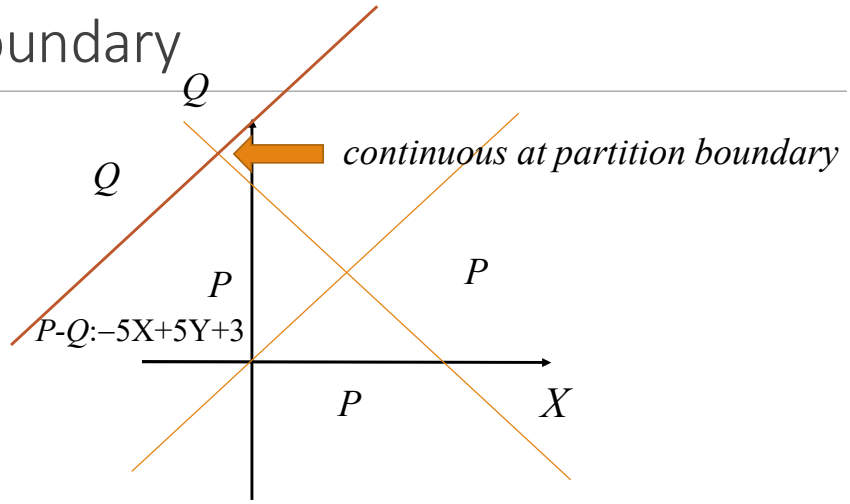

Class Boundaries are

- hyperplanes in the input space
- continuous at the partition boundary

Conclusions

- Partition and class boundaries are hyperplanes in the input space.
- These boundaries are continuous.
- The continuities limit the freedom of degree of the neural network

Thank you