Computing stable resultant-based minimal solvers by hiding a variable

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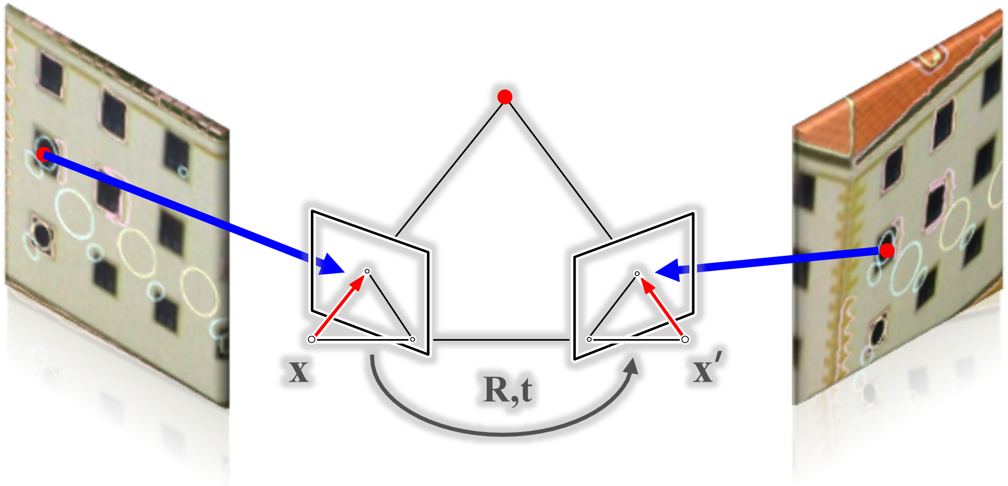
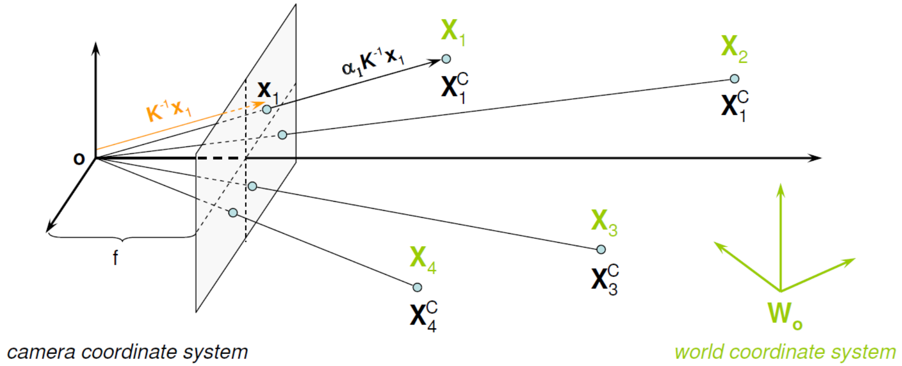
Motivation

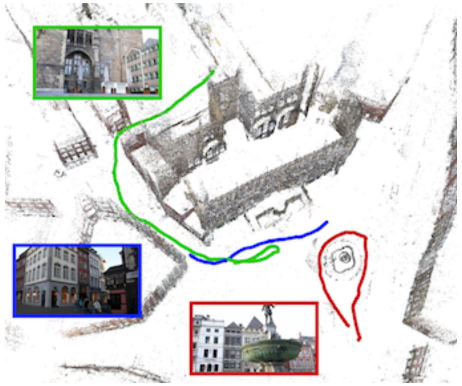
The estimation of the relative and absolute camera pose is a fundamental problem in computer vision





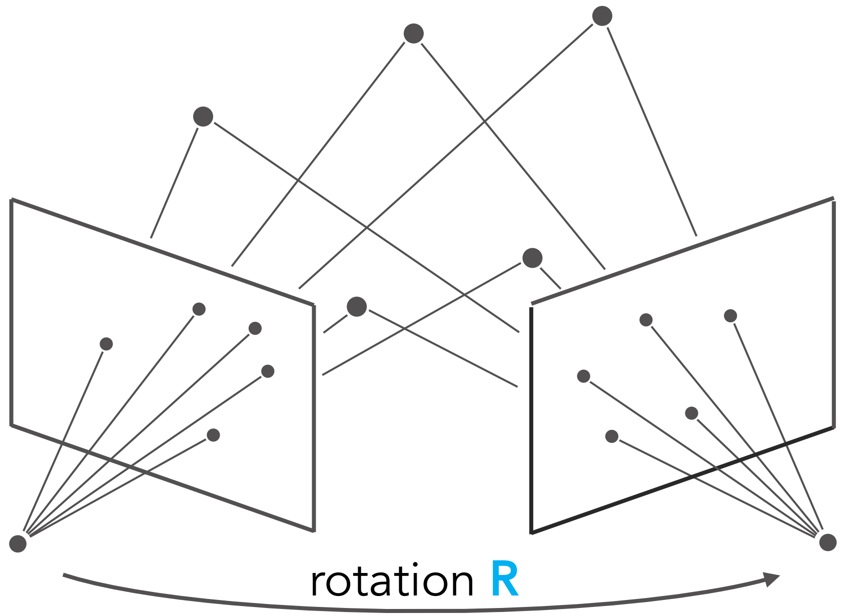
Applications:

- 3D reconstruction
- Localization
- SLAM
- AR & VR
- Autonomous Driving



Motivation

- High or real-time performance, robust and stable solutions.
- Contaminated input measurements $\longrightarrow$ problems have to be solved for multiple input samples (RANSAC)
- Processing time in RANSAC exponentially grows with the sample size
- Minimal number of point correspondences $\longrightarrow$ Minimal problems $\longrightarrow$ complex systems of polynomial equations



E.g. Relative pose between two calibrated cameras:

5 image point correspondences

$$\mathbf{x}_j'^{\top} \mathtt{E}\, \mathbf{x}_j = 0, \ j = 1, \dots, 5$$

$$\det(\mathtt{E}) = 0$$

$$2\left(\mathtt{E}\mathtt{E}^T\right)\mathtt{E} - trace(\mathtt{E}\mathtt{E}^T)\mathtt{E} = 0$$

Motivation

All "non-degenerate" instances of one problem result in systems of equations of the "same form" (same unknowns, same monomials, different coefficients)

$$\mathbf{x}_j'^{\top}\mathtt{E}\,\mathbf{x}_j = 0,\ \ j = 1,\dots,5$$
$$\det(\mathtt{E}) = 0$$
$$2\left(\mathtt{E}\mathtt{E}^T\right)\mathtt{E} - trace(\mathtt{E}\mathtt{E}^T)\mathtt{E} = 0$$

$$\Downarrow$$

Solve many instances of a given problem with the same system of polynomial equations, only with different coefficients.

Need to solve quickly and accurately.

State-of-the-art methods

- Creating specific efficient solvers
  - Can solve only systems of equations of a given form

- Grobner basis-based method [1,2]
  - Applicable to most minimal problems
  - Well explored and optimized
  - Automatic generator

- Sparse restultant-based method
  - Alternative method with comparable stability and efficiency [3]
  - Adding polynomial of special form $f_0 = x_i - \lambda$

[1] V. Larsson et al. Efficient solvers for minimal problems by syzygy-based reduction. In CVPR, 2017.
[2] V. Larsson et al. Beyond grobner bases: Basis selection for minimal solvers. In CVPR, 2018.
[3] S. Bhayani, Z. Kukelova, and J. Heikkila, "A sparse resultant based method for efficient minimal solvers," in CVPR, 2020.

Gröbner basis method

Offline preprocessing

Input equations

$$\begin{array}{c} f_1 \\ f_2 \\ f_3 \end{array}\begin{bmatrix} * & & * & & * & * & \\ & & * & & * & * & * \\ & * & & * & & * & * \end{bmatrix}\begin{bmatrix} xy \\ y^2z \\ xz \\ x \\ y \\ z \\ 1 \end{bmatrix} = 0$$

Elimination template matrix

$$\begin{array}{c} xyf_3 \\ yf_2 \\ xzf_1 \\ xf_1 \\ f_1 \\ f_2 \\ f_3 \end{array}\begin{bmatrix} * & & & * & & & & * & & * & \\ & * & & * & & * & & & * & & \\ & * & * & & * & & & * & & & \\ & & & * & & * & * & & * & & \\ & & & & * & & * & & * & * & \\ & & & & & & * & & * & * & * \\ & & & & & * & & * & & * & * \end{bmatrix}$$

Online solver

Gauss-Jordan Elimination

$$\begin{bmatrix} * & & & & & & & * & * & * & * \\ & * & & & & & & * & * & & * \\ & & * & & & & & * & * & * & * \\ & & & * & & & & * & * & * & * \\ & & & & * & & & * & * & * & * \\ & & & & & * & & * & * & * & * \\ & & & & & & * & * & & * & * \end{bmatrix}$$

Multiplication matrix

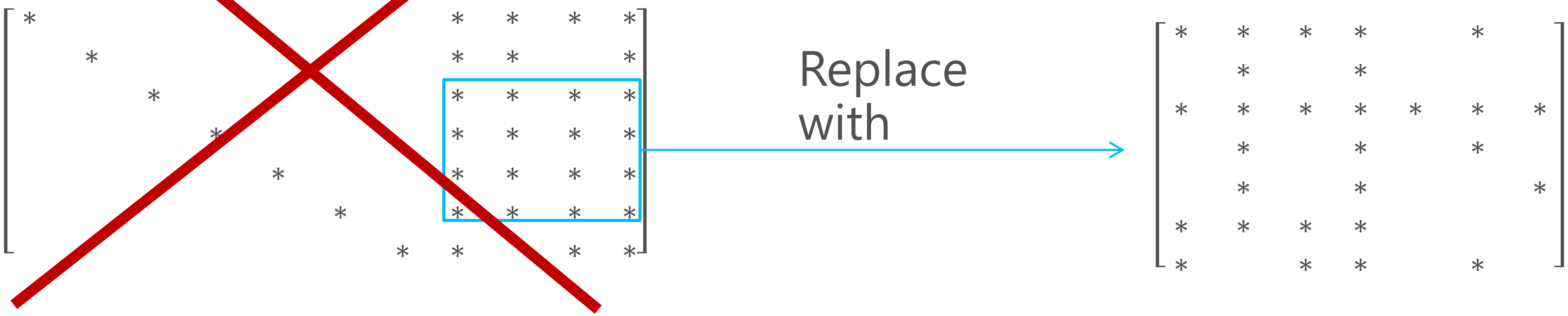
$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

Eigendecomposition → Solutions

Main idea

~~Gauss-Jordan Elimination~~

Larger matrix for Eigendecomposition



Can be numerically unstable

Eigendecomposition → Solutions

Hidden variable resultant method

Offline preprocessing

Input equations

$$\begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix}\begin{bmatrix} * & & * & & * & * & \\ & & * & & * & * & * \\ & * & & * & & * & * \end{bmatrix}\begin{bmatrix} xy \\ y^2z \\ xz \\ x \\ y \\ z \\ 1 \end{bmatrix} = 0$$

$\xrightarrow{\text{Hiding variable } z}$

Extended set of polynomials

$$\begin{matrix} xyf_3 \\ y^2f_2 \\ xf_1 \\ xf_2 \\ f_1 \\ f_2 \\ f_3 \end{matrix}\begin{bmatrix} * & * & & * & z^3* & & * \\ & * & z^2* & * & & * & \\ & * & & z* & * & z* & * \\ & & & * & z* & * & \\ * & & * & & * & & * \\ & z^3* & * & & * & & \\ * & & * & & z* & * & * \end{bmatrix}\underbrace{\begin{bmatrix} xy \\ y^2 \\ x^2 \\ xy \\ x \\ y \\ 1 \end{bmatrix}}_{b} = 0$$

Online solver

Fill template matrix

$$\mathbf{M}(z) = \begin{bmatrix} * & * & & * & z^3* & & * \\ & * & z^2* & * & & * & \\ & * & & z* & * & z* & * \\ & & & * & z* & * & \\ * & & * & & * & & * \\ & z^3* & * & & * & & \\ * & & * & & z* & * & * \end{bmatrix}$$

Sparse resultant constraint

$\longrightarrow$ $\mathbf{M}(z)b = 0$

$\det \mathbf{M}(z) = 0$ → Solutions to z

However determinant computation may be numerically unstable

Extracting the solutions

We rewrite it as a Polynomial Eigenvalue Problem (PEP) [1]

$$\mathbf{M}(z) = \mathbf{M}_0 + z\mathbf{M}_1 + \ldots + z^l\mathbf{M}_l$$

- Which can be solver as a Generalized Eigenvalue Problem (GEP) [1]

$$\mathbf{A}\, v = z\mathbf{B}v$$

$\Downarrow$

$\mathbf{A}$ and $\mathbf{B}$ contain only coefficients of original system

$\Downarrow$

Efficient tools for eigendecomposition $\longrightarrow$ Solutions

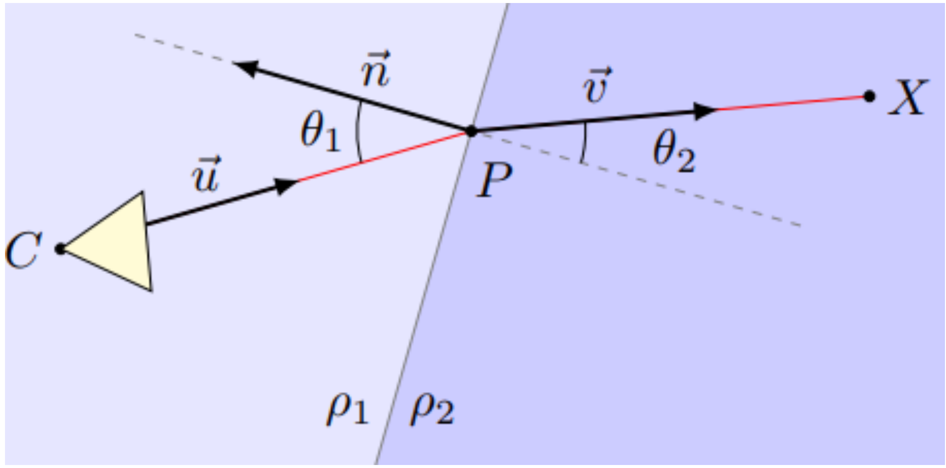
[1] J. Heikkilä. Using sparse elimination for solving minimal problems in computer vision. In ICCV, 2017.

Features of our proposed approach

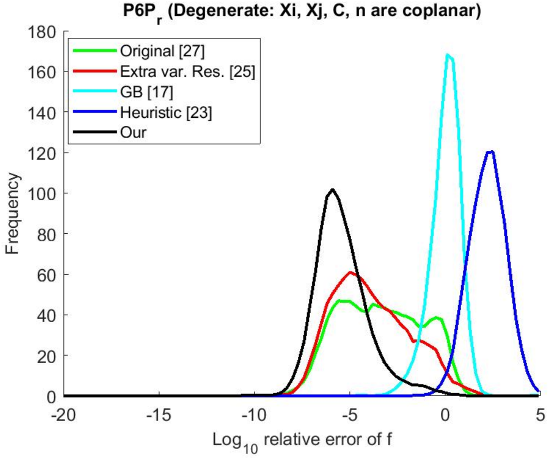
- Polytope based approach is used for extending the given polynomial system [1]
- Extensions and improvements to the approach by Heikkila [1]:
  - Testing polynomial combinations of all sizes $\longrightarrow$ reduction in solver size
  - Explicit test for rank $\longrightarrow$ accurate solver
  - A method to remove parasitic (zero) eigenvalues in the offline step,
    - Leading to improved solver size for some minimal problems
- Our proposed approach is applicable to many minimal problems
- Can be easily automated

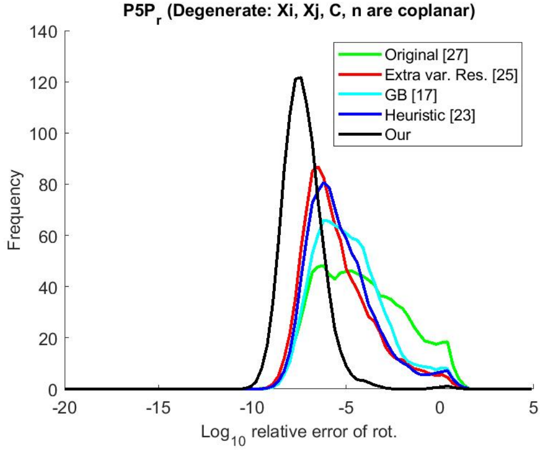
[1] J. Heikkilä. Using sparse elimination for solving minimal problems in computer vision. In ICCV, 2017.

Absolute pose estimation for refractive surfaces



Solver stability in close-to-degenerate scenes





Comparison of important computation steps of different solvers												
Comp. step	$\mathbf{P5P_r}$				$\mathbf{P6Pf_r}$				$\mathbf{P2P_r}$			
	GB[1]	Heur [2]	Res [3]	**Our**	GB[1]	Heur [2]	Res [3]	**Our**	GB[1]	Heur [2]	Res [3]	**Our**
G-J/QR	199 x 215	188 x 215	78 x 83	**-**	636 x 654	398 x 416	248 x 300	**-**	913 x 317	597 x 621	142 x 74	**-**
EIG	44 x 44	16 x 16	25 x 25	**-**	18 x 18	18 x 18	52 x 52	**-**	24 x 24	24 x 24	32 x 32	**-**
GEP	-	-	-	**36 x 36**	-	-	-	**110 x 110**	-	-	-	**124 x 124**
Time (ms)	0.4937	0.6683	0.3344	**0.4743**	4.6193	2.0822	1.53	**5.192**	10.5689	4.9556	0.7292	**6.5612**

More minimal problems

Problem	Comparison of important computation steps and stability of some more minimal problems													
	Our				GB [1]					Res [3]				
	Size	Stability			Size		Stability			Size		Stability		
	GEP	Mean	Median	Fail. (%)	GJ	Eig.	Mean	Median	Fail. (%)	GJ	Eig.	Mean	Median	Fail. (%)
Rolling shutter pose	20 x 20	-12.97	-13.09	0.6	47 x 55	8 x 8	-12.51	-12.70	**0**	47 x 55	8 x 8	-12.16	-12.34	**0**
Rel. pose 6pt. + sided rad. dist.	30 x 30	-12.47	-12.69	**0.2**	34 x 60	26 x 26	-11.42	-11.72	0.52	14 x 40	26 x 26	-11.65	-11.94	0.34
Abs. pose quiver	43 x 43	-12.00	-12.48	**0**	233 x 253	20 x 20	-11.18	-11.51	0.32	68 x 92	24 x 24	-12.39	-12.60	0.08
Rel. pose 9pt. + 2 rad. dist.	72 x 72	-12.96	-13.11	**0.1**	165 x 189	24 x 24	-9.81	-10.31	5.14	90 x 117	27 x 27	-9.81	-10.02	3.32
Rel. pose 7pt. + 1 sided rad. dist.	35 x 35	-12.31	-12.53	**0.02**	51 x 70	19 x 19	-10.57	-10.90	0.30	51 x 70	19 x 19	-10.71	-10.95	0.38

[1] V. Larsson et al. Efficient solvers for minimal problems by syzygy-based reduction. In CVPR, 2017.
[2] V. Larsson et al. Beyond grobner bases: Basis selection for minimal solvers. In CVPR, 2018.
[3] S. Bhayani, Z. Kukelova, and J. Heikkila, “A sparse resultant based method for efficient minimal solvers,” in CVPR, 2020.
[4] S. Haner and K. Åström. Absolute pose for cameras under flat refractive interfaces. In CVPR, 2015.

Conclusions

- We studied a hidden variable sparse resultant-based approach for solving minimal camera geometry problems in a stable and efficient way.
- Our proposed approach is an important alternative to the state-of-the-art Grobner basis-based approach for generating efficient minimal solvers.
- Our proposed approach may be useful for solving complex problems where the state-of-the-art solvers have numerical instabilities.

Thank You

For more details, visit our poster session, **PS3.11**

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