Computer Science and Engineering

Tensorized Feature Spaces for Feature Explosion

Ravdeep S Pasricha, Pravallika Devineni, Evangelos E. Papalexakis and Ramakrishnan Kannan

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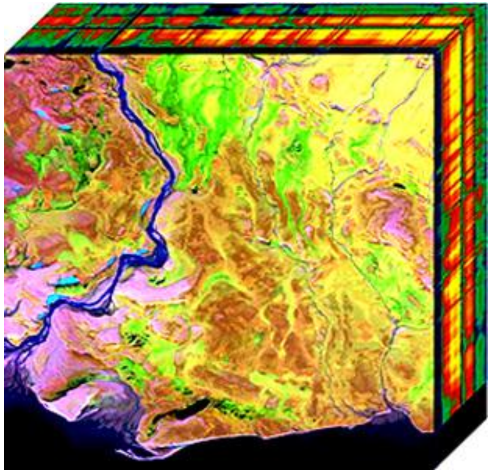
Roadmap

1. **Introduction**
2. Problem Formulation
3. Proposed Method
4. Experiments and Results
5. Conclusions

Hyperspectral Images(HSI)

- Images with hundreds of spectral bands at each pixel.
- Used in aerial land surveys with aircrafts or satellites.
- Different objects reflect different wavelengths of the spectral bands.

Example of 3-D Tensor:



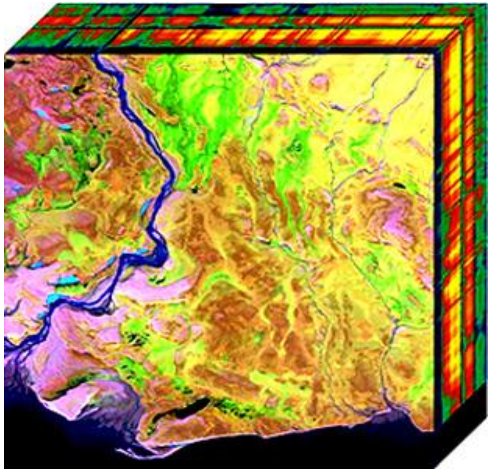
Different spectral bands

Image taken from: https://en.wikipedia.org/wiki/Hyperspectral_imaging

HSI(2)

- Each pixel has different features corresponding to spectral bands.
- Usually two type of tasks:
  - Pixel Classification task - Where each pixel belongs to one class.
  - Unmixing task - Composed of multiple materials.

Example of 3-D Tensor:

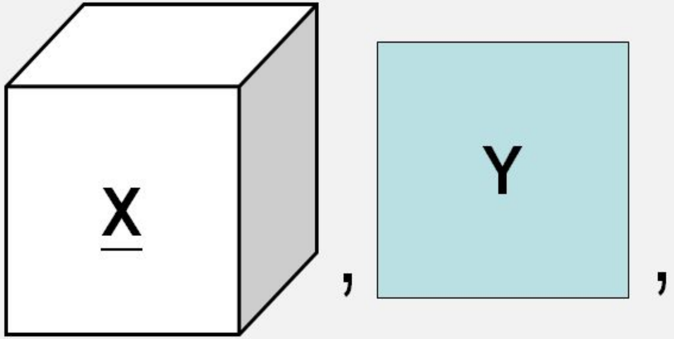


Different spectral bands

Image taken from: https://en.wikipedia.org/wiki/Hyperspectral_imaging

Problem:

Given:



HSI 3-D Tensor **Label Matrix**

, and Tensor Rank *R*

Generate a feature space for a classifier such that pixels in the image are classified into one of the given classes.

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Preliminary Definitions

- **Kronecker Product** of two matrices such that:

$$\mathbf{A} \in \mathbb{R}^{I \times J} \qquad \mathbf{B} \in \mathbb{R}^{M \times N}$$

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} \dots a_{1J}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} \dots a_{2J}\mathbf{B} \\ \vdots & \vdots \quad \ddots \\ a_{I1}\mathbf{B} & a_{I2}\mathbf{B} \dots a_{IJ}\mathbf{B} \end{bmatrix} \qquad \mathbf{A} \otimes \mathbf{B} \in \mathbb{R}^{IM \times JN}$$

Preliminary Definitions (2)

- **Khatri-Rao Product(KRP)** of two matrices is a column-wise Kronecker product.

$$\mathbf{A} \in \mathbb{R}^{I \times R} \qquad \mathbf{B} \in \mathbb{R}^{J \times R}$$

$$\mathbf{A} \odot \mathbf{B} = [\mathbf{a_1} \otimes \mathbf{b_1} \quad \mathbf{a_2} \otimes \mathbf{b_2} \quad \dots \quad \mathbf{a_R} \otimes \mathbf{b_R}]$$

$$\mathbf{A} \odot \mathbf{B} \in \mathbb{R}^{IJ \times R}$$

Preliminary Definitions (3)

- The **CP decomposition** of a 3-mode tensor of size $I \times J \times K$ for a particular rank R is given by sum of R rank-one tensors:

$$\underline{\mathbf{X}} \approx \sum_{r=1}^{R} \mathbf{A}(:, r) \circ \mathbf{B}(:, r) \circ \mathbf{C}(:, r)$$

A, B and **C** are the factor matrices.

A: $I \times R$, **B**: $J \times R$ and **C**: $K \times R$

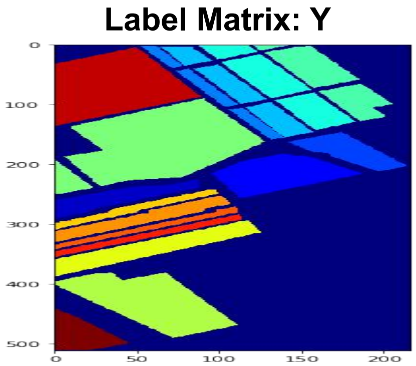
$\circ$ denotes the three way outer product.

- **Tensor Completion** is the task of predicting missing values in a tensor using tensor decomposition.

Problem Definition

Given:

- **$\underline{\mathbf{X}}$** : a 3-D HSI tensor of size $I \times J \times K$
- **Y :** a label matrix of size $I \times J$ and
- *R:* Tensor rank



Generate a feature space for a classifier such that pixels in the image are classified into one of the given classes.

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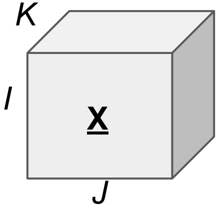
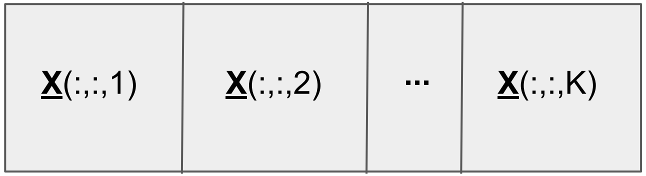
Proposed Method: ORION

- Intuition: Map the input space to higher dimensional space.
- CP decomposition of **X** of size $I \times J \times K$ yields 3 factor matrices.
    - **A**: $I \times R$, **B**: $J \times R$ and **C**: $K \times R$
- Tensorized feature space: Khatri-Rao Product of matrices **A** and **B**

$$\mathbf{A} \odot \mathbf{B} \in \mathbb{R}^{IJ \times R}$$

- Mode-3 matricization of tensor: $\mathbb{R}^{IJ \times K}$ ⇦ **Matrix Bounded by *K***



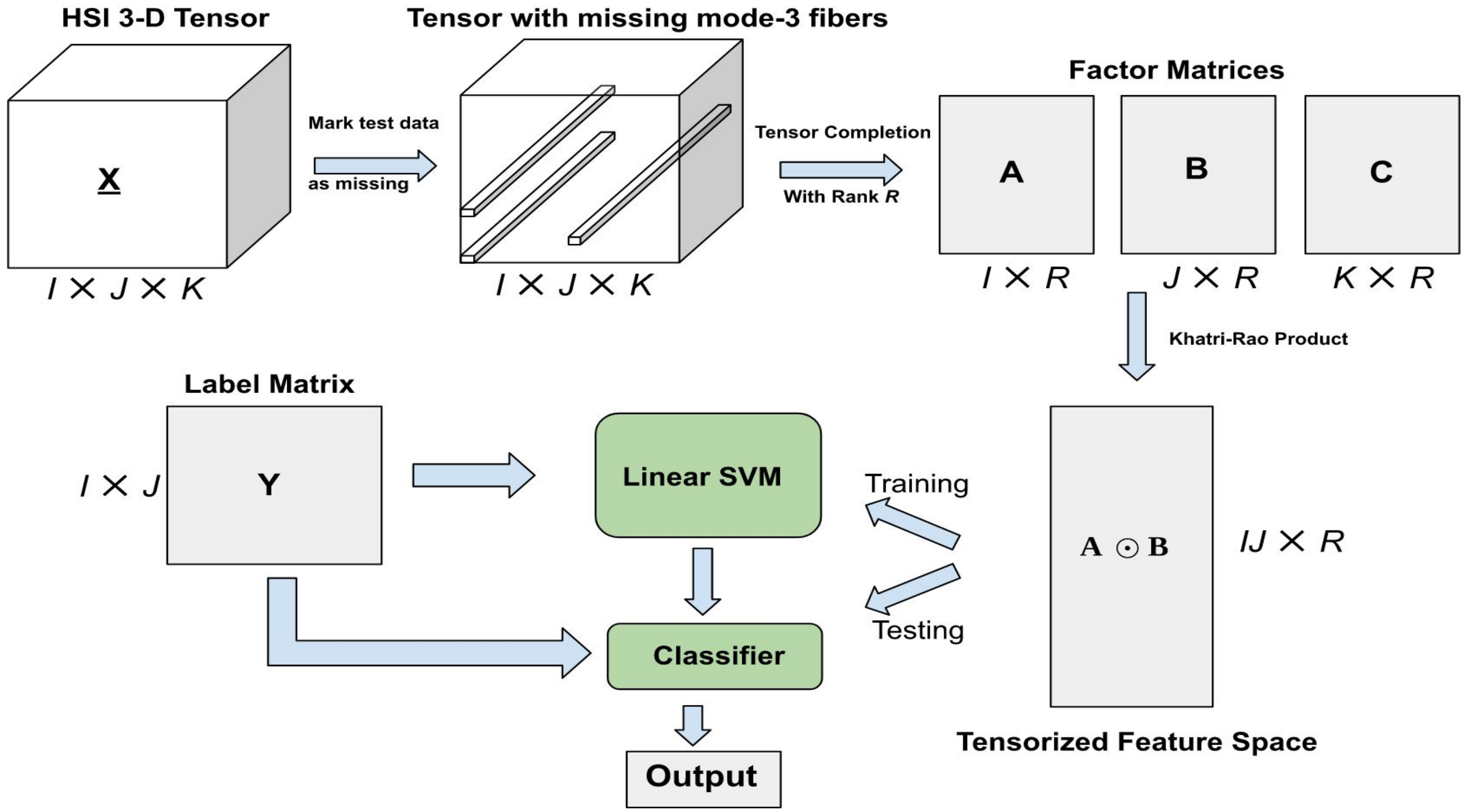


$\mathbf{X}_1$ of dimension $I \times JK$

Example of mode-1 matricization

- Tensor Rank for CP is bounded by min(IJ, JK, KI).

Proposed Method: ORION



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Experimental Evaluation

- Implemented Using Matlab and Python
  - Tensor Toolbox by Sandia Labs[1] for tensor completion task.
  - Scikit-Learn for classification task.
  - Tensorly[2] for tensor operations in Python.
- Datasets:
  - Indian Pines
  - University of Pavia
  - Salinas
  - Salinas-A (Subscene of Salinas dataset)

[1] http://www.sandia.gov/~tgkolda/TensorToolbox/index-2.6.html
[2] http://tensorly.org/stable/index.html

Experimental Evaluation(2)

- Baseline Methods:
  - Support Vector Machines(SVMs):
    - Linear Kernel
    - Polynomial Kernel
    - RBF Kernel
  - Multi-Layer Perceptron
- Evaluation Metrics:
  - Classification Accuracy
  - *F1* Score

Classification Accuracy (80-20)

TABLE II

CLASSIFICATION ACCURACY OF ALL THE METHODS FOR 80-20 SPLIT

	Indian Pines	Pavia University	Salinas-A	Salinas
Linear SVM	0.8708$\pm$ 0.0035	0.9176 $\pm$ 0.0017	**0.9986 $\pm$ 0.0016**	0.9339 $\pm$ 0.0014
Polynomial SVM	0.8979$\pm$ 0.0054	0.9481 $\pm$ 0.0015	0.9978 $\pm$ 0.0015	0.9463 $\pm$ 0.0014
RBF SVM	0.9178$\pm$ 0.0050	0.9622 $\pm$ 0.0020	**0.9985 $\pm$ 0.0017**	0.9620 $\pm$ 0.0024
MLP	0.9182$\pm$ 0.0057	**0.9635 $\pm$ 0.0041**	0.9982 $\pm$ 0.0010	0.9629 $\pm$ 0.0045
ORION -1000	**0.9916$\pm$ 0.0022**	0.9502 $\pm$ 0.0032	0.9690 $\pm$ 0.0067	**0.9927 $\pm$ 0.0010**
ORION -2000	**0.9949$\pm$ 0.0022**	**0.9828 $\pm$ 0.0030**	0.9680 $\pm$ 0.0063	**0.9954 $\pm$ 0.0006**

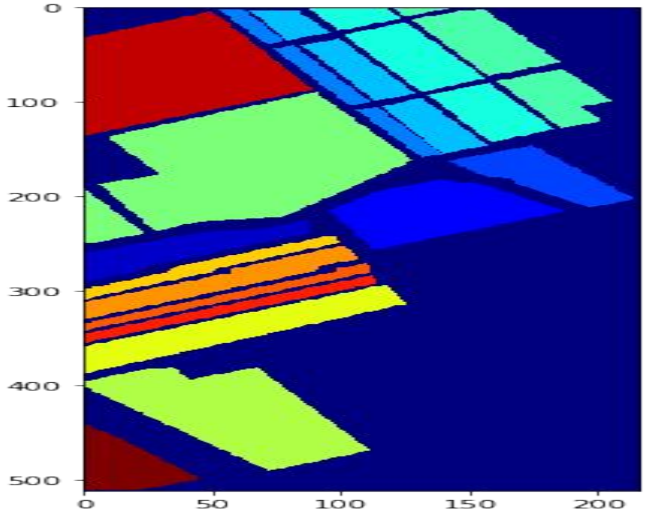
Classification Accuracy (30-70)

TABLE III

CLASSIFICATION ACCURACY OF ALL THE METHODS FOR 30-70 SPLIT

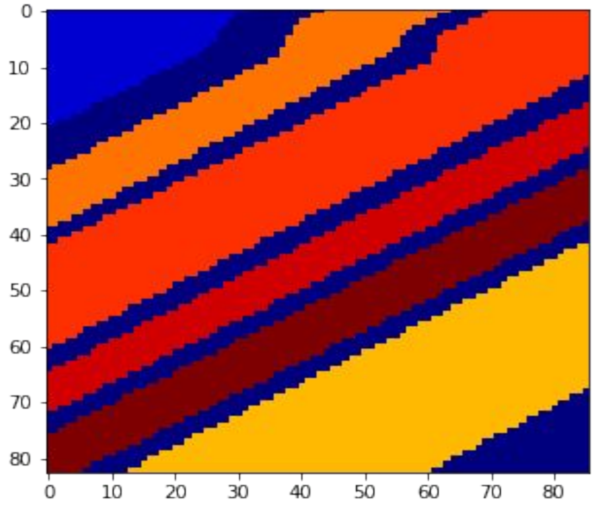
	Indian Pines	Pavia University	Salinas-A	Salinas
Linear SVM	0.8371 ± 0.0034	0.9134 ± 0.0015	$\mathbf{0.9965 \pm 0.0010}$	0.9322 ± 0.0007
Polynomial SVM	0.8511 ± 0.0042	0.9367 ± 0.0010	0.9941 ± 0.0017	0.9406 ± 0.0009
RBF SVM	0.8739 ± 0.0041	$\mathbf{0.9546 \pm 0.0007}$	$\mathbf{0.9966 \pm 0.0011}$	0.9515 ± 0.0012
MLP	0.8693 ± 0.0098	$\mathbf{0.9556 \pm 0.0029}$	0.9931 ± 0.0029	0.9475 ± 0.0041
ORION -1000	$\mathbf{0.9725 \pm 0.0032}$	0.9119 ± 0.0015	0.8607 ± 0.0146	$\mathbf{0.9662 \pm 0.0013}$
ORION -2000	$\mathbf{0.9806 \pm 0.0031}$	0.9544 ± 0.0021	0.8982 ± 0.0073	$\mathbf{0.9832 \pm 0.0013}$

Discussion about Salinas and Salinas-A



Salinas

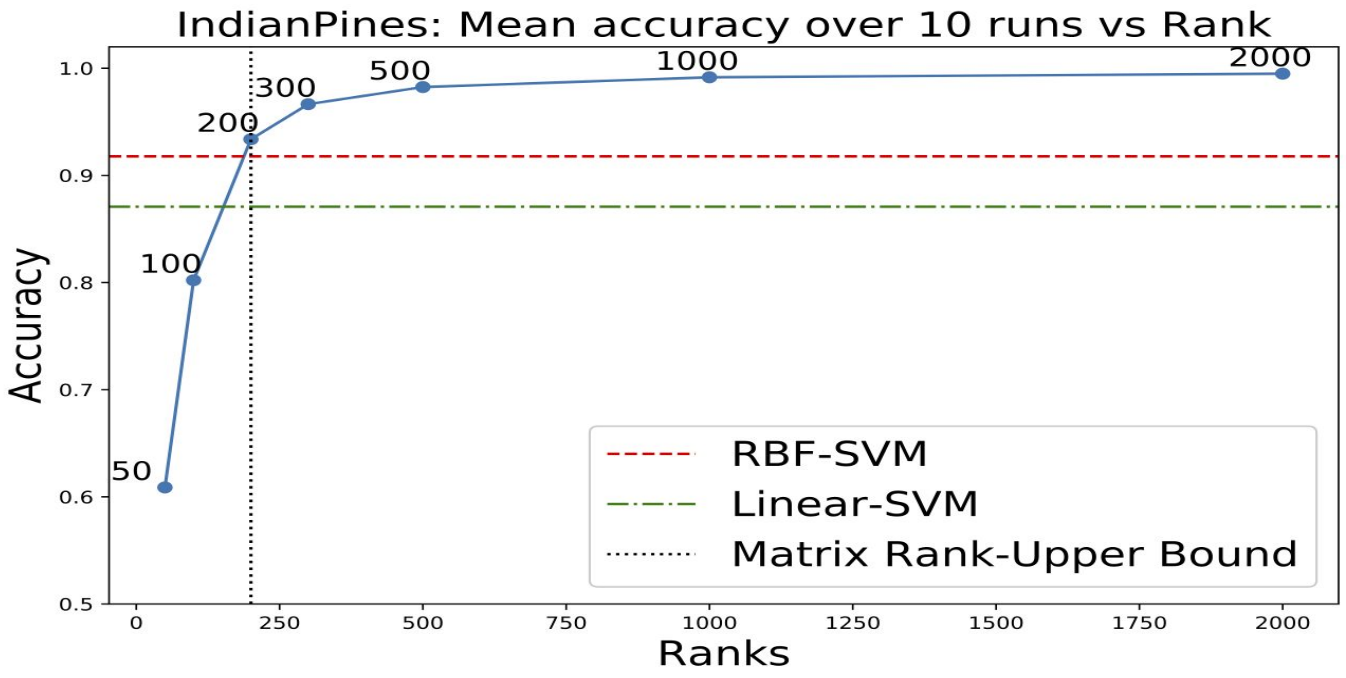
- Potentially has better trilinear structure



Salinas-A

- Linearly Separable

Classification Accuracy Vs Rank



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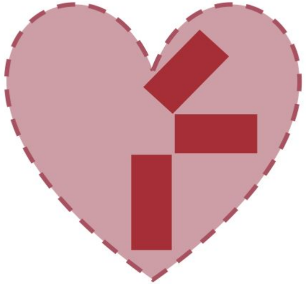
Conclusion

- Introduced tensorized feature space based on factors generated from tensor decomposition.
- Demonstrated effectiveness of our methods against traditional linear and non-linear supervised learning methods.

Our code is available at https://github.com/ravdeep003/ORION

Thank You!

I



Tensors

UCR Computer Science and Engineering

Oak Ridge National Laboratory

How to reach me:

- Email: rpasr001@ucr.edu
- LinkedIn: linkedin.com/in/ravdeep003
- Website: www.ravdeep.in
- Code: https://github.com/ravdeep003/ORION

M A D Lab @ UCR
Multi Aspect Data

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