

# Social Network Analysis using Knowledge-Graph Embeddings and Convolution Operations

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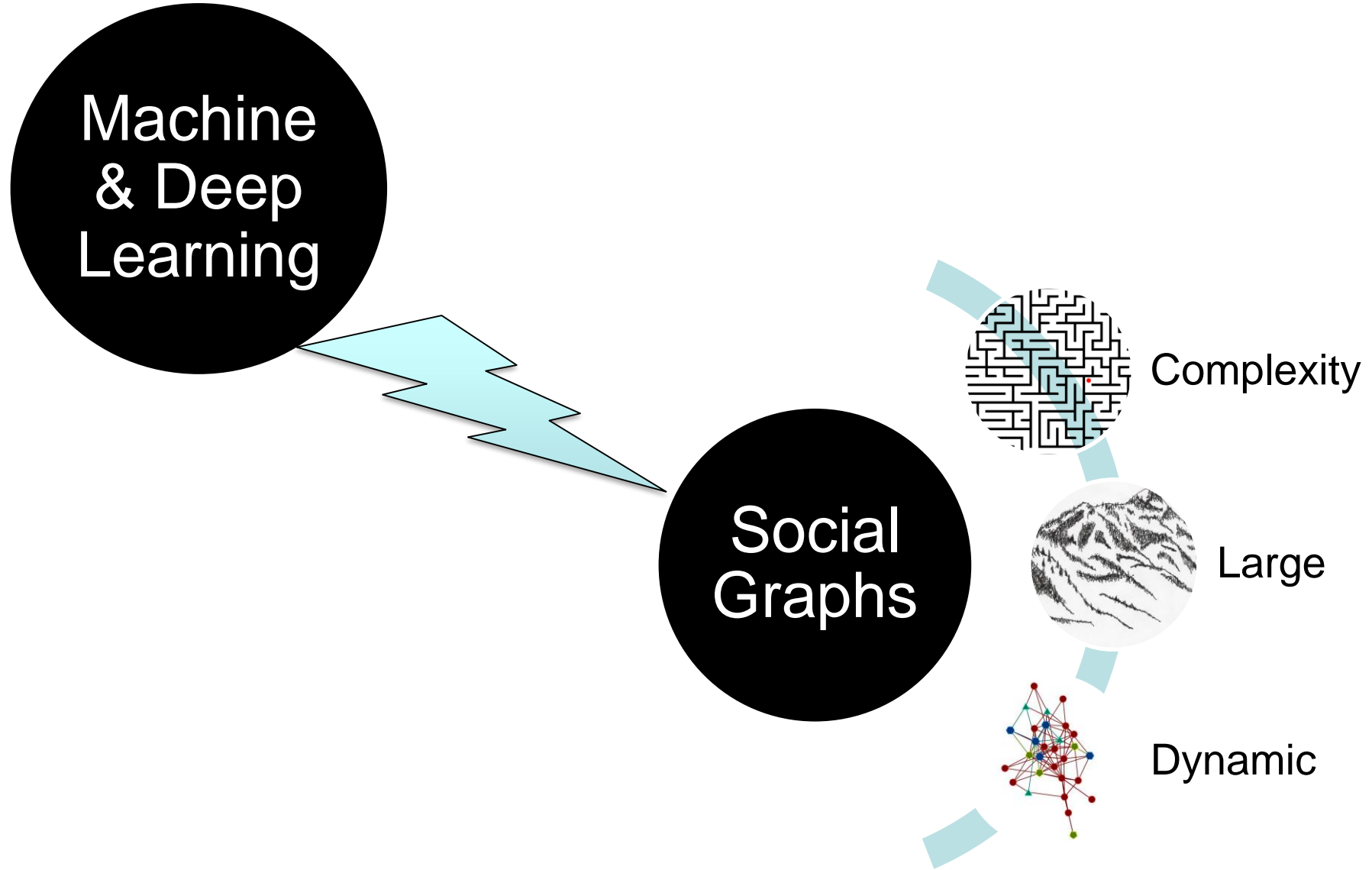
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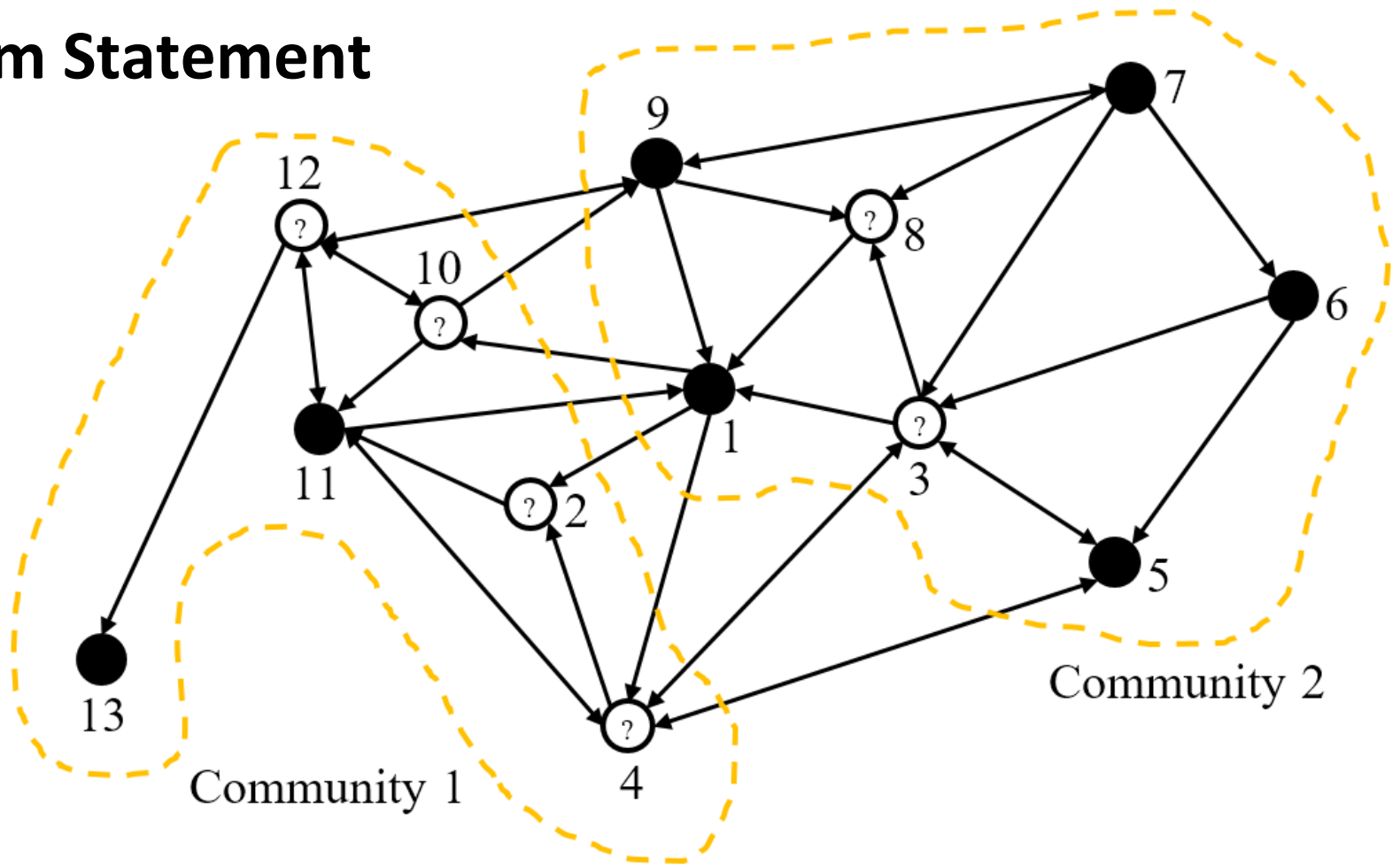
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# Introduction



# Problem Statement



$f_v^1$  ● Labelled Actor

$f_v^2$  ⊙ Unlabelled Actor

$f_v^3$  ⊙ Cluster Boundary

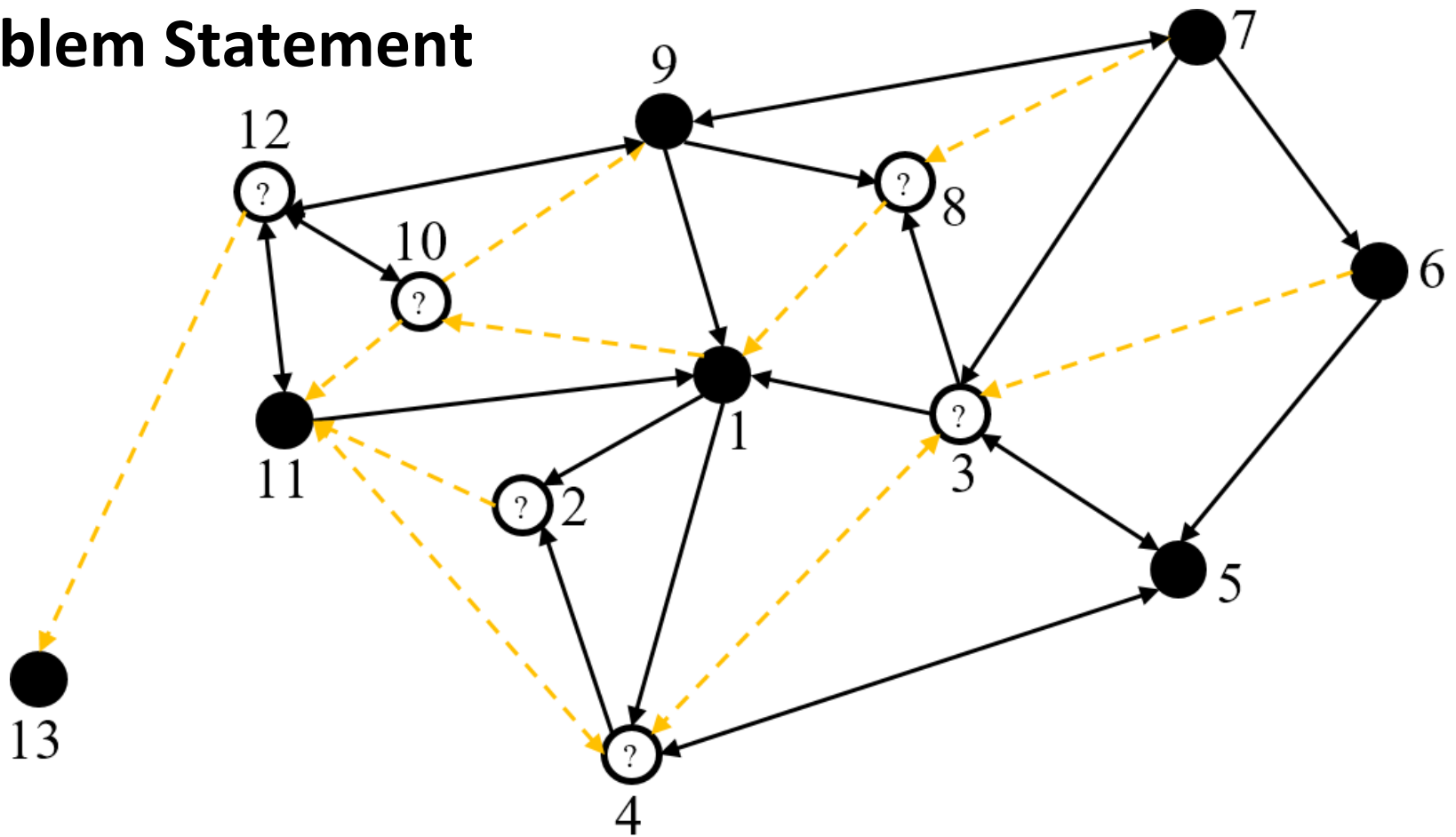
$f_v^4$  n Actor ID

$f_E^1$  ↔ Bidirectional Tie/Edge

$f_E^2$  → Unidirectional Tie/Edge



# Problem Statement



$f_v^1$  ● Labelled Actor

$f_v^2$  ⊙ Unlabelled Actor

$f_v^3$  n Actor ID

$f_E^1$  - - - Missing Tie

$f_E^2$  ↔ Bidirectional Tie/Edge

$f_E^3$  → Unidirectional Tie/Edge



# Problem Statement

## Social Network

$$SN = (V, E, f_V, f_E) \equiv (G, f_V, f_E)$$

$$V : |\{V\}| = M$$

set of actors/vertices with size, M

$$E : E \subset \{U \times V\} \subset \{V \times V\}$$

set of ties/edges between V (1)

$$f_V : V \rightarrow V'$$

vertices' metadata function

$$f_E : E \rightarrow E'$$

edges' metadata function

## Node Classification

$$V_{lbl} \subset V : V_{lbl} \rightarrow Y_{lbl}$$

partially labelled actors (or vertices)

$$V_{ulb} = V - V_{lbl}$$

unlabelled actors (or vertices) (2)

$$f : V \rightarrow Y$$

## Link Prediction

$$U \subset V : \{U|u_0, u_1, \dots, u_m\} \subset \{V|v_0, v_1, \dots, v_m\}$$

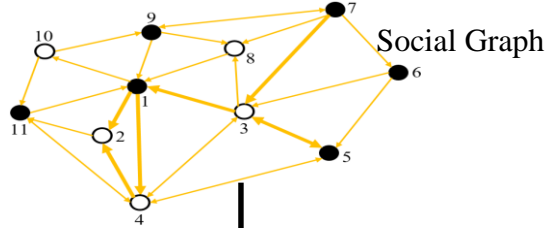
set of actors/vertices

$$E : (u_i, v_j) \in \{U \times V\}$$

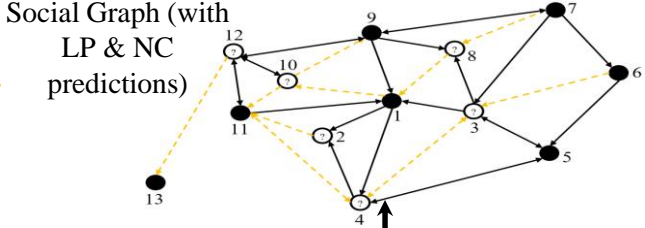
set of ties/edges (3)

$$f : \text{similarity}(U, V)$$





Input to RLVECN Framework



Pooled feature/activation maps alongside ground-truth labels,  $Y$ , are used for training a MLP classifier for LP and NC tasks

E <sup>+</sup> Ties		E <sup>-</sup> Ties	
U	V	U	V
0	5	0	1
0	6	0	2
$u_m$	$v_n$	$u_m$	$v_n$

**Preprocessing Layer** 1

Social graph is processed to generate E<sup>-</sup> (in addition to E<sup>+</sup>). Both E<sup>-</sup> and E<sup>+</sup> are represented via discrete values

**Embedding-Vector Layer** 2

Discrete representations of E<sup>-</sup> and E<sup>+</sup> are used to generate vector-space embedding, comprising 256 dimensions ( $\mathbb{R}^{256}$ ), per actor/node in E<sup>-</sup> and E

The vector-space embedding ( $\mathbb{R}^{256}$ ), of the constituent actors or nodes in SN (E<sup>-</sup> and E<sup>+</sup>), are used as input for generating pooled feature/activation maps via the ConvNet layer

**ConvNet or CNN Layer** 3

**ConvNet or CNN Layer** 3

**Classification Layer** 4



# Proposed (Formal) Methodology

## Knowledge-Graph (Vector) Embeddings RL-Layer:

$$\sum_{(u_i, v_j) \in E} \log Pr(u_i | v_j) = \sum_{(u_i, v_j) \in E} \log \frac{\exp(u_i \cdot v_j)}{\sum_{m=1}^M \exp(u_m \cdot v_j)} \quad (1)$$

## ConvNet Layer RL-Layer:

$$FeatureMap(F) = 1D\_InputMatrix(X) * Kernel(K)$$

$$f_i = (X * K)_i = (K * X)_i = \sum_{j=0}^{J-1} x_j \cdot k_{i-j} = \sum_{j=0}^{J-1} k_j \cdot x_{i-j} \quad (2)$$

$$r_i \in R = g(f_i \in F) = \max(0, F)$$

$$p_i \in P = h(r_i \in R) = \maxPool(R)$$

## Multi-Layer Perceptron (MLP) Classifier Layer:

$$Y = f_c(P, \Theta) \quad (3)$$



# Proposed Node-Classification Algorithm

**Input:**  $\{V, E, Y_{lbl}\} \equiv \{\text{Actors, Ties, Ground-Truth Labels}\}$

**Output:**  $\{Y_{ulb}\} \equiv \{\text{Predicted Labels}\}$

## Preprocessing:

$V_{lbl}, V_{ulb} \subset V = V_{lbl} \cup V_{ulb}$  //  $V_{lbl}$  : Labelled actors,  $V_{ulb}$  : Unlabelled actors

$E : (u_i, v_j) \in \{U \times V\}$  //  $(u_i, v_j) \equiv (\text{source, target})$

$E_{train} = E_t : u_i, v_j \in V_{lbl}$  //  $|E_{train}| = \sum indegree(V_{lbl}) + \sum outdegree(V_{lbl})$

$E_{pred} = E_p : u_i, v_j \in V_{ulb}$

$f_c \leftarrow \text{Initialize}$  // Construct classifier model

## Training:

**for**  $t \leftarrow 0$  **to**  $|E_{train}|$  **do**

$f : E_t \rightarrow [X_t \in \mathbb{R}^q]$  // Embedding operation

$f_t \in F = X_t \odot K_t$  // Convolution operation

$r_t \in R = g(F) = \max(0, f_t)$

$p_t \in P = h(R) = \maxPool(r_t)$

$f_c | \Theta : p_t \rightarrow Y_{lbl}$  // MLP classification operation

**end for**

**return**  $Y_{ulb} = f_c(E_{pred}, \Theta)$





# Proposed Link-Prediction Algorithm

**Input:**  $\{V, E, \mathbb{B}_{gTruth}\} \equiv \{\text{Actors, Ties, Ground-Truth Entities}\}$

**Output:**  $\{\mathbb{B}_{pred}\} \equiv \{\text{Predicted Entities}\}$

## Preprocessing:

$\mathbb{B}_{gTruth} : \{0, 1\} \equiv \{C0 : \text{-ve/False tie, } C1 : \text{+ve/True tie}\}$

$E = E_{+ves} \cup E_{-ves}$

$E : (u_i, v_j) \in \{U \times V\} \subset \{V \times V\}$  //  $\{V \times V\} \equiv \text{Set of all possible ties}$

$E_{train} = E_t : E \rightarrow \mathbb{B}_{gTruth}$  //  $|E_{train}| = E - E_{pred} = \text{Ground-Truth edgelist}$

$E_{pred} = E - E_{train}$  //  $E_{pred} : E'_{train} = \text{Complement of } E_{train}$

$f_c \leftarrow \text{Initialize}$  // Construct prediction model

## Training:

**while**  $E_{train} \neq \text{NULL}$  **do**

$f : E_t \rightarrow [X_t \in \mathbb{R}^q]$  // Embedding operation

$f_t \in F = X_t \odot K_t$  // Convolution operation

$r_t \in R = g(F) = \max(0, f_t)$

$p_t \in P = h(R) = \maxPool(r_t)$

$f_c | \Theta : p_t \rightarrow \mathbb{B}_{gTruth}$  // MLP :  $\Theta = \text{similarity}()$

**end while**

**return**  $\mathbb{B}_{pred} = f_c(E_{pred}, \Theta)$



# Experiments and Results (Node Classification)

	Cora		CiteSeer		Facebook	
	F1	RO	F1	RO	F1	RO
RLVECN	<b>0.83</b>	<b>0.91</b>	<b>0.73</b>	<b>0.85</b>	<b>0.87</b>	<b>0.98</b>
GCN	0.81	0.84	0.65	0.76	Vector features absent	
Node2Vec	0.69	0.80	0.45	0.68	0.81	0.87
DeepWalk	0.58	0.74	0.40	0.65	0.76	0.84
LINE	0.34	0.60	0.22	0.54	0.59	0.72
SDNE	0.33	0.60	0.23	0.55	0.55	0.71

	PubMed		Internet-Industry		Terrorists-Relation	
	F1	RO	F1	RO	F1	RO
RLVECN	<b>0.80</b>	<b>0.94</b>	<b>0.57</b>	<b>0.80</b>	0.83	0.96
GCN	Vector features absent				<b>0.84</b>	<b>0.97</b>
DeepWalk	0.50	0.64	0.49	0.65	0.80	0.87
Node2Vec	0.37	0.56	0.42	0.59	0.79	0.86
LINE	0.34	0.53	0.25	0.50	0.76	0.83
SDNE	0.32	0.55	0.25	0.50	0.76	0.82



# Experiments and Results (Link Prediction)

$C0 : \mathbb{B} = 0$  (-ve/False tie)

	Cora		CiteSeer		Internet-Industry		Facebook		PubMed		Terrorists-Relation	
	F1	RO	F1	RO	F1	RO	F1	RO	F1	RO	F1	RO
RLVECN	<b>0.98</b>	<b>0.98</b>	<b>0.99</b>	<b>0.99</b>	<b>0.85</b>	0.89	0.89	<b>0.99</b>	<b>0.99</b>	<b>1.00</b>	0.90	<b>0.99</b>
DistMult	0.91	0.93	0.94	0.94	0.80	<b>0.90</b>	0.89	0.98	0.92	0.96	0.84	0.97
ComplEx	0.90	0.92	0.93	0.94	0.79	<b>0.90</b>	0.86	0.97	0.91	0.95	0.77	0.95
ConvKB	0.89	0.91	0.93	0.93	0.79	<b>0.90</b>	0.82	0.93	0.87	0.91	0.71	0.93
HolE	0.81	0.84	0.82	0.85	0.79	0.85	<b>0.92</b>	0.98	0.94	0.95	<b>0.95</b>	<b>0.99</b>

$C1 : \mathbb{B} = 1$  (+ve/True tie)

	Cora		CiteSeer		Internet-Industry		Facebook		PubMed		Terrorists-Relation	
	F1	RO	F1	RO	F1	RO	F1	RO	F1	RO	F1	RO
RLVECN	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>0.95</b>	<b>0.90</b>	<b>0.98</b>	<b>1.00</b>	<b>0.99</b>	<b>1.00</b>	<b>0.99</b>	<b>1.00</b>
DistMult	0.92	0.93	0.94	0.94	0.89	<b>0.90</b>	0.97	0.98	0.95	0.96	0.97	0.97
ComplEx	0.91	0.92	0.93	0.94	0.88	<b>0.90</b>	0.97	0.97	0.94	0.95	0.95	0.95
ConvKB	0.90	0.91	0.92	0.93	0.88	<b>0.90</b>	0.95	0.95	0.92	0.93	0.93	0.93
HolE	0.89	0.84	0.87	0.85	0.91	0.85	<b>0.98</b>	0.98	0.97	0.95	<b>0.99</b>	0.99

