



# Quaternion Capsule Networks

Barış Özcan, Furkan Kınlı, Furkan Kıraç Özyeğin University

## Problem

- CNNs require many samples for a specific object to generalize to geometric variations such as novel viewpoints.
- The intrinsic structure of an object is not modelled in the architecture in terms of part-whole relationships.
- This deficiency is proven by the experiments in novel viewpoint setup of smallNORB dataset.
- Capsule Networks address this problem however, using matrix transformations have several disadvantages.

## Capsule Networks

- Aims to achieve generalization by modelling the hierarchical structure of parts for object classes in the data.
- Capsule is an encapsulation of multiple neurons allows to flow complex information between layers (i.e. pose, feature vector).
- Each part capsule votes for parent capsules pose by transforming part capsules pose. These transformations are learned during training.
- Parent capsule pose and activations are calculated via clustering-like routing mechanisms over the incoming votes. (i.e. EM-Routing, dynamic routing).

## Our assumption



- We constraint the pose representation to 3D rotations between capsules since convolutional connections are translation invariant.
- In the object coordinate system the intrinsic rotations  $w'_{ij}$  and  $w_{ij}$  between parts and whole is constant from any viewpoint. Thus if we can learn this rotation from the data the network should generalize to novel viewpoints.

## Quaternions and 3D rotation

- 4-dimensional complex numbers that represents efficient rotation computation.
- 3D rotation is represented with quaternion product.
  - Rotated quaternion(pose) is a pure quaternion.
  - Rotating quaternion(learned) is a unit quaternion



## Advantages of quaternion rotations

- Quaternions do not suffer from gimbal lock.
- Rotation matrices must be orthogonal which is harder than normalizing quaternions to ensure a proper rotation.
- Using quaternions reduces the number of parameters from 9 to 4 for each represented rotation in the network.

## Quaternion Capsules

- Each capsule is represented as a pure quaternion.
- Each connection is a learned quaternion rotation.
- Routing is via EM-Routing.
- Learned quaternions are normalized similarly to weight normalization technique.
- Each capsule contains its pose and activation.

## Network architecture



# Generalization to novel viewpoints experiment

- Training and test samples are from different viewpoints of the classes in smallNORB dataset.
- Azimuth experiment: training and test set are divided w.r.t. azimuth angles
- Elevation experiment: training and test set are divided w.r.t. elevations
- Training is stopped where training performances are similar for each model to test generalization fairly.

Train samples



#### Test samples



## Results on novel viewpoints

Viewpoints	Azimuth (%)							
(Models)	QCN	EM*	VB	EM	CNN			
Novel	7.5	13.4	11.3	13.5	20.0			
Familiar	3.7	3.7	3.7	3.7	3.7			
Viewpoints	Elevation (%)							
(Models)	QCN	EM*	VB	EM	CNN			
Novel	11.5	15.8	11.6	12.3	17.8			
Familiar	4.4	4.0	4.3	4.3	4.3			

### Results on common datasets

Models	smallNORB		MNIST		FashionMNIST		SHVN		CIFAR-10	
	Error (%)	# of Params	Error (%)	# of Params	Error (%)	# of Params	Error (%)	# of Params	Error (%)	# of Params
EM [7]	1.8	$\sim 310 K$	0.44	_	-	_	_	_	11.9	_
EM-IBM [37]	4.6	~335K	1.23 <sup>‡</sup>	$\sim 337 K^{\ddagger}$	10.44 <sup>‡</sup>	$\sim 337 K^{\ddagger}$	-	_	-	_
VB [8]	1.6	$\sim 169 K$	_	_	5.2	$\sim 172 K$	3.9	$\sim$ 323K	11.2	$\sim$ 323K
EM*	3.40	$\sim$ 317K	0.89	~319K	9.74	$\sim$ 319K	8.19	$\sim$ 320K	17.76	$\sim 460 \mathrm{K}$
QCN	2.29	$\sim \! 188 K$	0.37	$\sim \! 187 K$	6.92	$\sim \! 187 K$	4.63	$\sim \! 189 K$	13.92	$\sim \! 189 K$

## Conclusion

- QCNs generalize better to novel viewpoints in smallNORB.
- QCNs reduce the parameters by a large margin in Capsule Layers with rotation.
- Branching before extracting capsules adds more flexibility to the architecture.

## Thank You!

Official implementation in Pytorch is available at <a href="https://github.com/Boazrciasn/Quaternion-Capsule-Networks.git">https://github.com/Boazrciasn/Quaternion-Capsule-Networks.git</a>