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A Heuristic-Based Decision Tree for Connected Components Labeling of 3D Volumes

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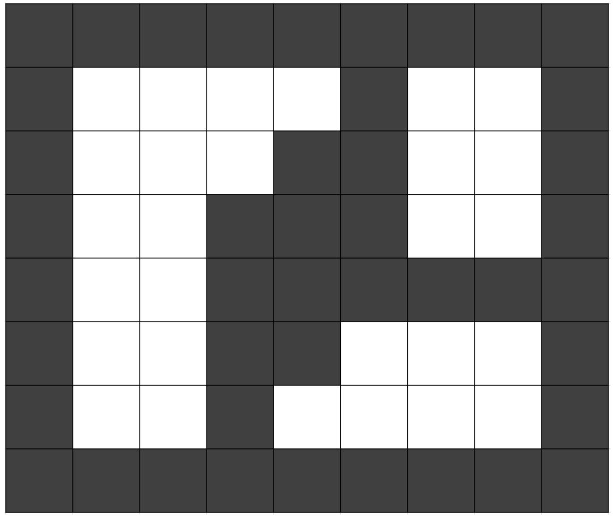
Connected Components Labeling (CCL)

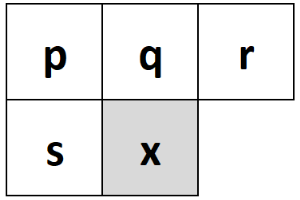
- Find all connected, foreground pixel regions within a binary image
- Each pixel region, or **connected component**, receives a unique label
- Fundamental for image segmentation and object recognition
- CCL should be as <u>fast</u> as possible



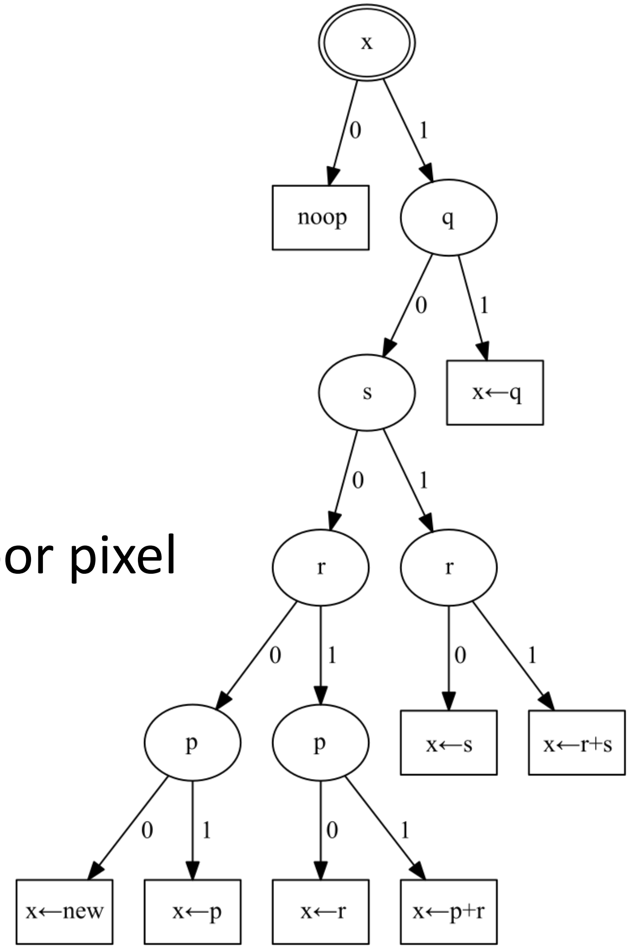
0	0	0	0	0	0	0	0	0
0	1	1	1	1	0	2	2	0
0	1	1	1	0	0	2	2	0
0	1	1	0	0	0	2	2	0
0	1	1	0	0	0	0	0	0
0	1	1	0	0	3	3	3	0
0	1	1	0	3	3	3	3	0
0	0	0	0	0	0	0	0	0

History of CCL Research (1/2)

- Rosenfeld and Pfaltz[1] invented **two scans** algorithms
  - First scan: gives each pixel a **provisional label** based on the neighborhood mask, and **solves label equivalences**
  - Second scan: assigns **definitive labels**
- Wu et al.[2] proposed **Optimal Decision Trees (ODTs)**
  - Label for a pixel can be decided **without reading** every neighbor pixel
  - Optimal binary decision tree minimizes pixel reads
  - Each tree node represents a pixel read



Rosenfeld mask



Optimal binary decision tree (ODT)
for the Rosenfeld mask

[1]A. Rosenfeld and J. L. Pfaltz, "Sequential Operations in Digital Picture Processing," *Journal of the ACM*, vol. 13, no. 4, pp. 471–494, Oct. 1966.
[2]K. Wu, E. Otoo, and K. Suzuki, "Two Strategies to Speed up Connected Component Labeling Algorithms," Lawrence Berkeley National Laboratory, Tech. Rep. LBNL-59102, 2005.

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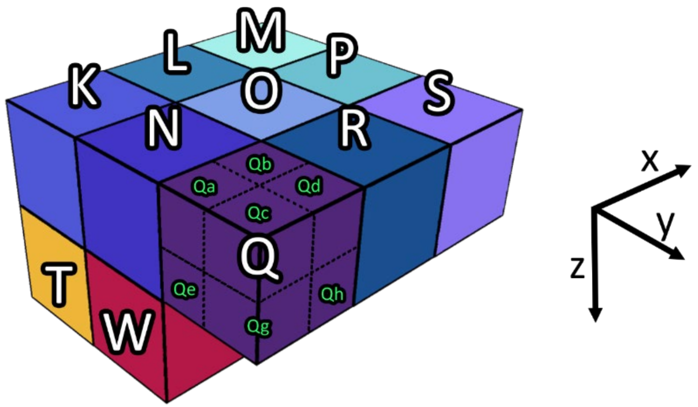
History of CCL Research (2/2)

- Grana et al.[1] proposed **block-based scan mask**
  - In a 2x2 block, all foreground pixels share the same label
  - Generated the ODT **automatically** since it is unfeasible by hand: 2^{16} cases, 136 nodes

(a)	b	c	d	e	(f)
g	h	i	j	k	(l)
m	n	o	p		
(q)	r	s	t		

2x2 Grana mask

- What about 3D CCL?
  - Multiple possible block-based masks: 2x1x1, 2x2x1 and 2x2x2
  - Explosion in computational complexity makes the ODT generation **infeasible**
  - **No existing 3D CCL algorithm employs a block-based mask**
  - Goal: generate a **near-optimal tree** with a heuristic strategy



2x2x2 voxel mask

[1]C. Grana, D. Borghesani, and R. Cucchiara, “Optimized Block-based Connected Components Labeling with Decision Trees,” *IEEE Transactions on Image Processing*, vol. 19, no. 6, pp. 1596–1609, 2010.

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Heuristics – Concept

- **Shannon Entropy** (information theory)
  - Given a set of events E, with P_i being the probability of an event $i \in E$, the entropy H_E is:

    $$H_E = \sum_i -P_i \log P_i$$

  - Entropy describes the uncertainty of outcomes

- **Decision Tree Learning**
  - Generate decision trees for complex datasets quickly
  - **Recursively partition** the dataset through entropy calculation
    1. Try *splitting* on every attribute
    2. Calculate **Information Gain (IG)** on subsets – IG measures average entropy reduction
    3. Apply *split* with the highest information gain
  - For CCL, the dataset is the decision table

							assign				merge	
x	p	q	r	s	no action	new label	x = p	x = q	x = r	x = s	x = p + r	x = r + s
0	-	-	-	-	1							
1	0	0	0	0		1						
1	1	0	0	0			1					
1	0	1	0	0				1				
1	0	0	1	0					1			
1	0	0	0	1						1		
1	1	1	0	0			*1*	1				
1	1	0	1	0							1	
1	1	0	0	1			*1*			1		
1	0	1	1	0				1	*1*			
1	0	1	0	1				1		*1*		
1	0	0	1	1								1
1	1	1	1	0			*1*	1	*1*			
1	1	1	0	1			*1*	1		*1*		
1	1	0	1	1							*1*	1
1	0	1	1	1				1	*1*	*1*		
1	1	1	1	1			*1*	1	*1*	*1*		

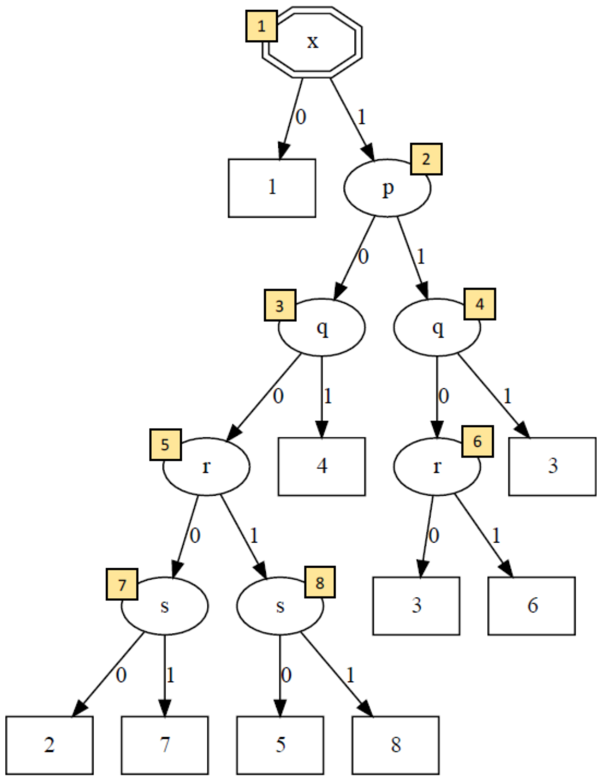
OR-decision table for the Rosenfeld mask

Applying Decision Tree Learning to CCL

- **Entropy Partitioning Decision Tree (EPDT)** generated for the Rosenfeld mask
- For each node, the pixel with maximum IG is chosen

Node	Depth	$H(S)$	p			q			r			s			x		
			H_0	H_1	IG	H_0	H_1	IG	H_0	H_1	IG	H_0	H_1	IG	H_0	H_1	IG
1	0	2.2	2.0	1.4	0.5	2.3	1.5	0.3	1.9	2.1	0.2	2.1	2.1	0.1	**0.0**	**2.4**	**1.0**
2	1	2.4	**2.0**	**0.8**	**1.0**	2.5	1.0	0.7	1.8	2.3	0.4	2.2	2.2	0.2			
3	2	2.0				**2.0**	**0.0**	**1.0**	1.5	1.5	0.5	1.5	1.5	0.5			
4	2	0.8				**1.0**	**0.0**	**0.3**	0.0	1.0	0.3	0.8	0.8	0.0			
5	3	2.0							**1.0**	**1.0**	**1.0**	1.0	1.0	1.0			
6	3	1.0							**0.0**	**0.0**	**1.0**	1.0	1.0	0.0			
7	4	1.0										**0.0**	**0.0**	**1.0**			
8	4	1.0										**0.0**	**0.0**	**1.0**			

- The result is **near-optimal**
  - Only one more node than the optimal decision tree
- Next step: apply EPDT to 3D block-based masks



EPDT for the Rosenfeld mask

3D EPDT algorithms

- New 3D EPDT CCL algorithms
- Varying block size and number of pixels
- **EPDT_19c**
  - Block size 2x1x1
  - Smallest 3D block-based mask
- **EPDT_22c**
  - Block size 2x1x1
  - Add borders pixels, for more efficient actions
- **EPDT_26c**
  - Block size 2x2x1
  - Largest tree that compilers can handle

Ka	**Kb**	**La**	**Lb**	**Ma**	Mb
Na	**Nb**	**Oa**	**Ob**	**Pa**	Pb
Qa	**Qb**	**Ra**	**Rb**	**Sa**	Sb

Ta	**Tb**	**Ua**	**Ub**	**Va**	Vb
Wa	**Wb**	**Xa**	**Xb**		

EPDT_19c 3D mask

Ka	**Kb**	**La**	**Lb**	**Ma**	Mb
Na	**Nb**	**Oa**	**Ob**	**Pa**	**Pb**
Qa	**Qb**	**Ra**	**Rb**	**Sa**	Sb

Ta	**Tb**	**Ua**	**Ub**	**Va**	Vb
Wa	**Wb**	**Xa**	**Xb**		

EPDT_22c 3D mask

Ka	Kb	La	Lb	Ma	Mb
Kc	**kd**	**Lc**	**Ld**	**Mc**	Md
Na	**Nb**	**Oa**	**Ob**	**Pa**	Pb
Nc	**Nd**	**Oc**	**Od**	**Pc**	Pd
Qa	**Qb**	**Ra**	**Rb**	**Pa**	Pb
Qc	Qd	Rc	Rd	Pc	Pd

Ta	Tb	Ua	Ub	Va	Vb
Tc	**Td**	**Uc**	**Ud**	**Vc**	Vd
Wa	**Wb**	**Xa**	**Xb**		
Wc	**Wd**	**Xc**	**Xd**		

EPDT_26c 3D mask

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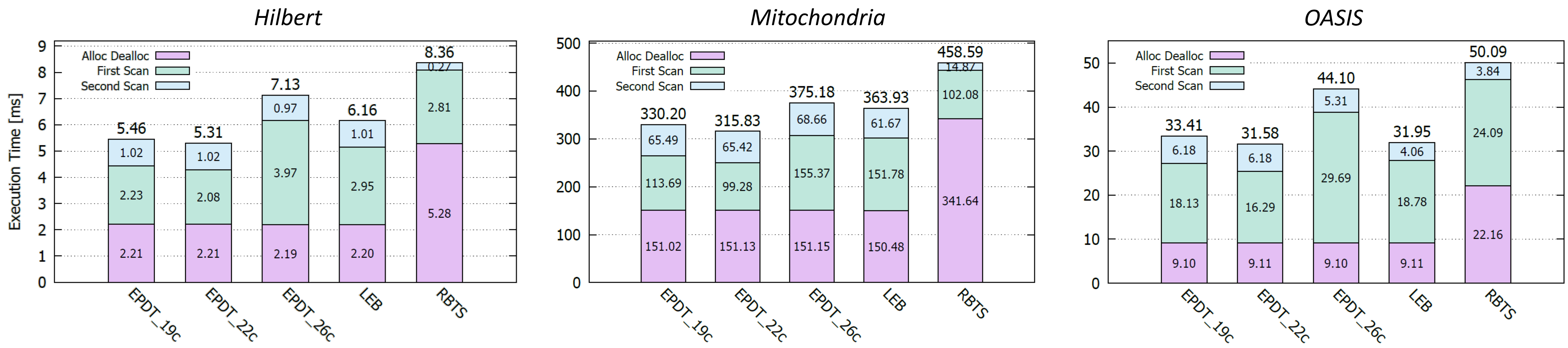
Experimental Results



- EPDT algorithms improve the performance of the first scan by saving many **memory accesses**
- EPDT_26c has a very large decision tree → bad impact on instruction cache
- EPDT_22c improves current state-of-the-art[1]

Algorithm	Binary Image	Labels Image	Equivalences Vector	Total
LEB	11.461	27.182	9.851	48.494
EPDT_19c	14.917	17.760	1.169	33.846
EPDT_22c	14.057	17.753	1.145	32.955
EPDT_26c	13.695	13.145	0.728	27.568

Average number of load/store operations on the OASIS dataset, expressed in millions.

[1]L. He, Y. Chao, and K. Suzuki, "Two Efficient Label-Equivalence-Based Connected-Component Labeling Algorithms for 3-D Binary Images," *IEEE Transactions on Image Processing*, vol. 20, no. 8, pp. 2122–2134, 2011.

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