# **AVAE:** Adversarial Variational Auto Encoder

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# Introduction

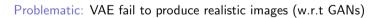
## Two main types of generative models

• VAEs have several advantages over GANs

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GAN	VAE	
+ realistic images	+ disentangled latent space	
<ul> <li>mode collapse</li> </ul>	+ encoder model	
<ul> <li>difficult to invert</li> </ul>	+ easy to train	
	<ul> <li>blurry images</li> </ul>	



- ► How can we explain this lack of realism ?
- Can we combine the best of VAEs and GANs ?

# Understanding VAEs and GANs

## Which problem for VAEs to produce realistic images ?

1. Information bottleneck:

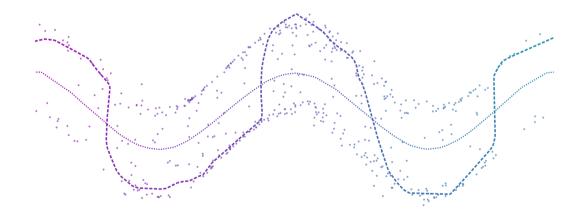
$$\mathcal{L}_{\mathsf{VAE}} = \underbrace{\mathbb{E}\left[\mathbb{E}_{q_{\theta_e}(z|x)}\left[-\log p_{\theta_d}\left(x|z\right)\right]\right]}_{\mathsf{reconstruction error}} + \underbrace{I_{\theta}(x;z)}_{\mathsf{mutual information}} + \underbrace{\mathsf{KL}(p_{\theta_e}(z)||p(z))}_{\mathsf{prior on } z}$$
(1)

- $\rightarrow\,$  incomplete information
- $\rightarrow\,$  mean value of all possible images
- $\rightarrow$  blurry results
- 2. Underestimation of natural image manifold dimensionality:
  - ightarrow approximation of the manifold with a simpler one
  - $\rightarrow$  uncertainty on other dimensions responsible of smaller variations (e.g. textures)
  - $\rightarrow\,$  mean value of all possible images
  - $\rightarrow$  blurry results (no texture in images)

GANs also underestimate the dimension of the natural image manifold.

- $\rightarrow$  Question: How are they able to produce realistic images ?
- $\rightarrow$  Answer: Mode collapse !  $\rightarrow$  only a few but plausible texture configurations are generated.

#### Illustration on a toy example



dots: data points

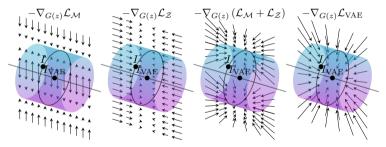
dotted line: VAE manifold

dashed line: GAN manifold

### How to solve the VAE problem ?

Objective: Create a reconstruction error  $\mathcal{L}_{\mathcal{Z}}$ :

- that is powerful enough to favor accurate reconstructions.
- that does not favor blurry reconstruction to allow realistic reconstructions.



- cylinders: real data high-dimensional manifold
- black line: low-dimensional manifold of VAEs reconstructions
- arrows: gradient of different losses

With reconstruction errors of the form  $\mathcal{L}_{\mathcal{Z}}(\hat{x}, x) = \frac{1}{2} ||f(\hat{x}) - g(x)||^2$  where:

- f is an arbitrary differentiable function
- g is a stochastic function

Optimal solutions  $\hat{x}^*(z)$  verifies:

$$f(\hat{x}^*(z)) = \mathbb{E}_{g(x) \sim \rho_{\theta_e}(g(x)|z)}[g(x)]$$
(2)

- $f(\hat{x})$  should carry the maximum of information about  $\hat{x}$  and g(x) should be close to f(x).
- Common optimum with the GAN objective  $\iff p(f(\hat{x}^*(z))) = p(f(x))$  for  $z \sim p(z)$  and  $x \sim p_D(x)$ .

$$\mathcal{L}_{\mathcal{Z}}(\hat{x},x) = \textit{MSE}(\hat{x},x) = rac{1}{2} ||\hat{x} - x||^2 o \mathsf{optimal}$$
 solution:

$$\hat{x}^*(z) = \mathbb{E}_{x \sim \rho_{\theta_e}(x|z)}[x]$$
(3)

- $f(\hat{x})$  carry all the information about  $\hat{x}$  as it is the identity, and g(x) = f(x).
- Optimal solution = mix of likely solutions  $\rightarrow$  blurry / unrealistic image.  $p(f(\hat{x}^*(z))) = p(\hat{x}^*(z)) \neq p(x) = p(f(x))$  for  $z \sim p(z)$  and  $x \sim p_D(x)$ .

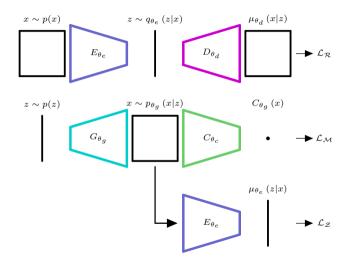
The AVAE framework

With:  

$$\begin{cases}
f(\hat{x}) = \frac{\mu_{\theta_e}(\hat{x})}{\sigma_{\theta_e}} \\
g(x) = \frac{\sqrt{1 - \sigma_{\theta_e}^2}}{\sigma_{\theta_e}} z & \rightarrow \\
\end{cases}
\qquad \mathcal{L}_{\mathcal{Z}}(\hat{x}) = \frac{1}{2} \left\| \frac{\mu_{\theta_e}(x) - \sqrt{1 - \sigma_{\theta_e}^2} z}{\sigma_{\theta_e}} \right\|^2$$
(4)

- $f(\hat{x})$  carry the information about  $\hat{x}$  contained in z, and  $g(x) = \frac{\sqrt{1-\sigma_{\theta_e}^2}}{\sigma_{\theta_e}} z \approx \frac{\mu_{\theta_e}(x)}{\sigma_{\theta_e}} = f(x)$
- $\mu_{\theta_e}(\hat{x}^*(z)) = \sqrt{1 \sigma_{\theta_e}^2} z \rightarrow p(\mu_{\theta_e}(\hat{x}^*(z))) = \mathcal{N}(\mu_{\theta_e}(x); 0, I \Sigma) = p(\mu_{\theta_e}(x)).$

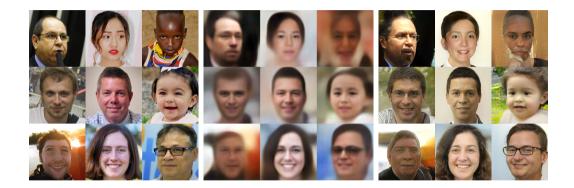
### Full AVAE framework



## Results

metric	VAE	GAN	VAE/GAN	BiGAN	Ours
mse ↓	$0.03\pm0.00$	—	$0.07\pm0.00$	$0.18\pm0.01$	$0.05\pm0.00$
Ipips $\downarrow$	$0.18\pm0.00$	—	$0.09\pm0.00$	$0.16\pm0.00$	$0.11\pm0.00$
fid $\downarrow$	$60.04 \pm 0.47$	$14.54\pm0.41$	$26.45\pm4.66$	$18.49\pm5.06$	$15.01\pm0.82$

- MSE: favorable to VAE a priori.
- MSE: favorable to our approach a priori.
- LPIPS & FID: favorable to VAE/GAN a priori.



original images

VAE decoder reconstructions

generator reconstructions

# Conclusion