A Spectral Clustering on Grassmann Manifold via Double Low Rank Constraint

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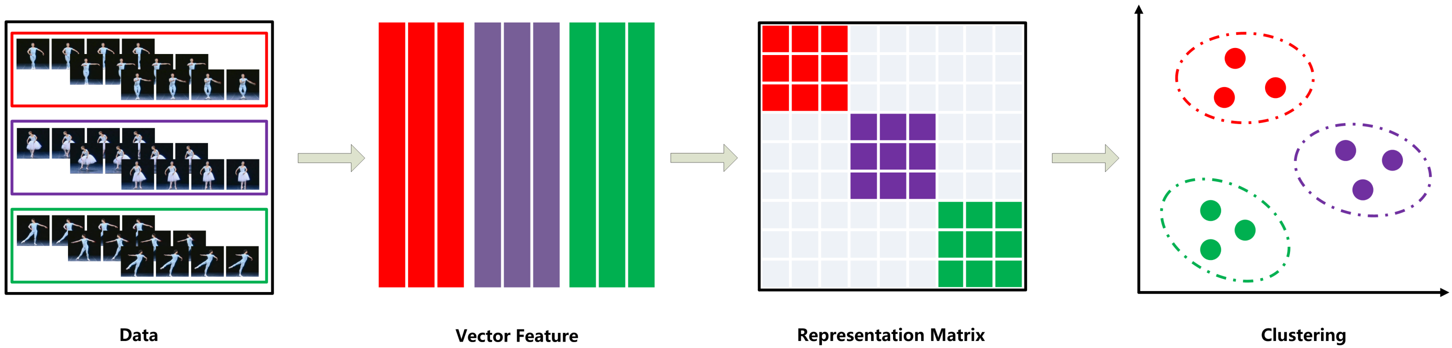
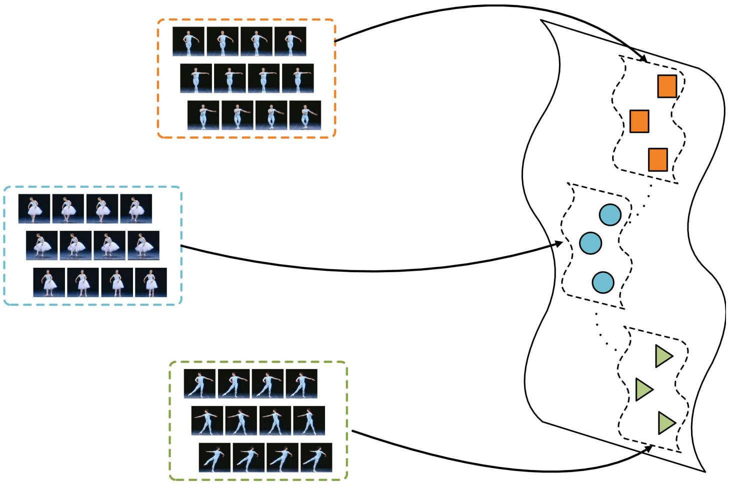
Background



Low rank and spectral clustering method:

- Learning representation matrix from data based on low rank representation
- Using NCut or K-Means method for clustering results
- Low rank representation based clustering method with linear Euclidean distance (ICML 2010, ICCV 2011)

Background



Grassmann Manifold:

- Constructing the Grassmann manifold feature for video or imageset data
- The set of Grassmann manifold data has non-linear structure
- Low rank representation based clustering method on Grassmann manifold with non-linear distance (ACCV 2014, ACM TKDD 2018)

Nuclear norm based low rank constraint:

$$\|\mathbf{X}\|_* = \sum_{i=1}^{n} \sigma_i(\mathbf{X})$$

- This low rank constraint would deviate the optimal solution and lead to suboptimal solution
- Traditional nuclear norm treats all singular values equally and prefers to punish the larger singular values than the small ones
- Smaller singular values always represent noise and would decrease the clustering accuracy

Contributions

The main contributions of our paper are following:

- Proposing a novel low rank representation model on Grassmann for video and imageset data clustering;
- Nuclear norm and bilinear factorization are combined as double low rank representation to reveal low-rank property more exactly;
- An effective algorithm is proposed to solve the complicated optimization problem of the proposed model.

Objective Function Formulation

Traditional low rank representation model:
For a set of Grassmann manifold samples $\mathcal{Y} = \{\mathbf{Y}_1, \mathbf{Y}_2, ..., \mathbf{Y}_n\}$, we could formulate the low rank based clustering model with traditional convex nuclear norm:

$$\min_{\mathbf{X}} \quad \lambda \|\mathbf{X}\|_* + \sum_{i=1}^{n} \|\mathbf{Y}_i \ominus \biguplus_{j=1}^{n} \mathbf{Y}_j \circledast x_{ji}\|_{\mathcal{M}},$$

where the first term is the low rank constraint, the second term is the reconstructed term for Grassmann manifold samples, and $\mathbf{X} \in \mathbb{R}^{n \times n}$ represents the coefficient matrix.

Objective Function Formulation

Double Low Rank Constraint:

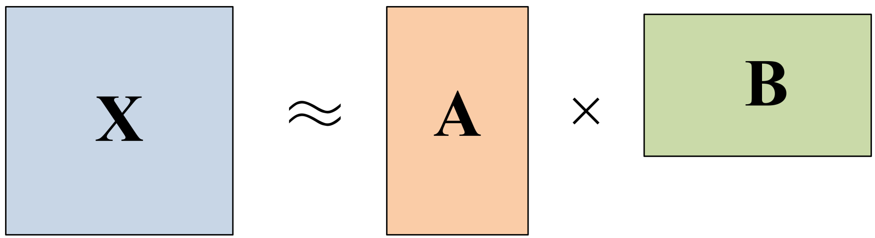


Figure 1: The illustration of bilinear representation.

$$\begin{aligned} \min_{\mathbf{X},\mathbf{A},\mathbf{B}} \quad & \lambda\|\mathbf{X}\|_* + \sum_{i=1}^{n} \|\mathbf{Y}_i \ominus \biguplus_{j=1}^{n} \mathbf{Y}_j \circledast x_{ji}\|_{\mathcal{G}} + \alpha(\|\mathbf{A}\|_F^2 + \|\mathbf{B}\|_F^2), \\ \text{s.t.} \quad & \mathbf{X} = \mathbf{A}\mathbf{B}, \end{aligned}$$

where $\mathbf{A} \in \mathbb{R}^{n\times r}$, $\mathbf{B} \in \mathbb{R}^{r\times n}$, and r is the expected rank of matrix $\mathbf{X}$. We call this model as the Double Low Rank constraint based clustering model on Grassmann manifold (G-DLR)

Objective Function Formulation

Nonlinear Metric for Grassmann manifold data:

$$\text{dist}_g(\mathbf{Y}_1, \mathbf{Y}_2) = \frac{1}{2}\|\Pi(\mathbf{Y}_1) - \Pi(\mathbf{Y}_2)\|_F,$$

where $\|\mathbf{X}\|_F = \sqrt{\sum_{i=1,j=1}^{n} x_{ij}^2}$ represents the Frobenius norm, $\Pi(\cdot)$ is a mapping function defined as below:

$$\Pi : \mathcal{G}(p, m) \longrightarrow Sym(m), \Pi(\mathbf{Y}) = \mathbf{Y}\mathbf{Y}^T,$$

where $Sym(m)$ represents the m-dimension symmetric matrix space.

Objective Function Formulation

The optimization:

$$\begin{aligned}\min_{\mathbf{X},\mathbf{A},\mathbf{B}} \quad & \lambda\|\mathbf{X}\|_* + \sum_{i=1}^{n} \|\mathbf{Y}_i \ominus \biguplus_{j=1}^{n} \mathbf{Y}_j \circledast x_{ji}\|_{\mathcal{G}} + \alpha(\|\mathbf{A}\|_F^2 + \|\mathbf{B}\|_F^2), \\ \text{s.t.} \quad & \mathbf{X} = \mathbf{AB}.\end{aligned}$$

By denoting $g_{ij} = \mathrm{tr}((\mathbf{Y}_j^T \mathbf{Y}_i)(\mathbf{Y}_i^T \mathbf{Y}_j))$ and $\mathbf{G} = \{g_{ij}\}_{n\times n} \in \mathbb{R}^{n\times n}$, we have:

$$\begin{aligned}\min_{\mathbf{X},\mathbf{A},\mathbf{B}} \quad & \lambda\|\mathbf{X}\|_* + \mathrm{tr}(\mathbf{X}^T\mathbf{G}\mathbf{X}) - 2\mathrm{tr}(\mathbf{G}\mathbf{X}) + \alpha(\|\mathbf{A}\|_F^2 + \|\mathbf{B}\|_F^2). \\ \text{s.t.} \quad & \mathbf{X} = \mathbf{AB}.\end{aligned}$$

Experimental Datasets

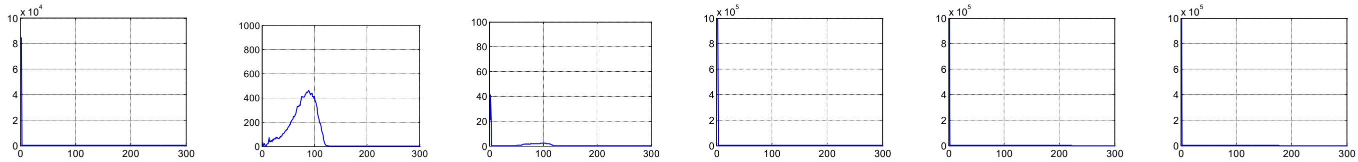
(a) (b)

(c)

Figure 2: Three datasets for experiments: (a) Extended Yale B dataset; (b) CMU-PIE face dataset; (c) Traffic dataset.

Experimental Results

Convergence analysis



(a) (b) (c) (d) (e) (f)

Figure 3: The convergence curves of G-DLR on Extended Yale B dataset.

Results analysis:

The clustering results are measured by the clustering Accuracy (ACC).

Table 1: The accuracy results of various methods on three datasets.

Method	SSC	LRR	LS3C	SCGSM	G-KM	G-CLRSR	G-LRR	G-PSSVLR	G-DNLR	G-DLR
Extended Yale B	0.4032	0.4659	0.2461	0.7946	0.8365	0.8194	0.8135	0.9035	0.9749	**0.9957**
CMU-PIE	0.5231	0.4034	0.2761	0.5732	0.6025	0.6289	0.6153	0.6213	0.6618	**0.7219**
Traffic	0.5920	0.6822	0.7169	0.6562	0.7470	0.8438	0.8132	0.8592	0.8300	**0.8774**
avg.	0.5061	0.5172	0.4130	0.6747	0.7287	0.7640	0.7473	0.7947	0.8222	**0.8650**

Conclusion

- Proposing a new low rank representation model on Grassmann manifold for video or imageset clustering task;
- In the proposed model, reweighted approach and non-convex function are combined to seek a better low-rank representation matrix instead of the traditional convex nuclear norm;
- An effective algorithm is proposed to solve the complicated optimization problem of the proposed model.

Thank You!

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