Learning Connectivity with Graph Convolutional Networks

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Outline

- Introduction
- Learning connectivity in GCNs
- Experiments
- Conclusion

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Motivation

- Graph convolutional networks (GCNs) aim at generalizing deep learning to arbitrary irregular domains.
- Existing spatial GCNs follow a neighborhood aggregation scheme, and its success is reliant on the topology (structure) of input graphs.
- However, graph structures (either available or handcrafted) are powerless to optimally capture all the relationships between nodes as their setting is oblivious to the targeted applications.
- E.g., node-to-node relationships, in human skeletons, capture the intrinsic anthropometric characteristics of individuals (useful for their identification) while other connections, yet to infer, are necessary for recognizing their dynamics and actions.

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Contribution : learning connectivity in GCNs

- We introduce a novel framework that learns convolutional filters on graphs together with their topological properties.
- The latter are modeled through matrix operators that capture multiple aggregates on graphs, learned using a constrained cross-entropy loss.
- We consider different *constraints* (including stochasticity, orthogonality and symmetry) acting as regularizers.
- Stochasticity implements random walk Laplacians while orthogonality models multiple aggregation operators with non-overlapping supports ; it also avoids redundancy and oversizing the learned GCNs with useless parameters. Symmetry reduces further the number of training parameters.

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Problem statement

- Considering $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ endowed with $\{s(u)\}_u$ and (ii) $\mathbf{A}$,

$$(\mathcal{G} \star \mathcal{F})_{\mathcal{V}} = f\big(\mathbf{A}\ \mathbf{U}^\top\ \mathbf{W}\big).$$

- Here $\mathbf{A}\mathbf{U}^\top$ acts as a feature extractor which collects non-differential and differential statistics including means and variances of node neighbors, before applying convolutions.
- We use the chain rule in order to derive the gradient $\frac{\partial E}{\partial \mathbf{vec}(\mathbf{A})}$ and update $\mathbf{A}$ using SGD ; we upgrade the latter to learn both the convolutional parameters $\mathbf{W}$ together with $\mathbf{A}$ while implementing *orthogonality, stochasticity and symmetry*.
- Orthogonality allows us to design $\mathbf{A}$ with a minimum number of parameters, stochasticity normalizes nodes by their degrees and allows learning random walk Laplacians, while symmetry reduces further the number of training parameters.

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Stochasticity

- Stochasticity requires adding equality and inequality constraints in SGD, i.e., $\mathbf{A}_{ij} \in [0,1]$ and $\sum_q \mathbf{A}_{qj} = 1$.
- We consider a reparametrization of the learned matrices, as $\mathbf{A}_{ij} = h(\hat{\mathbf{A}}_{ij}) / \sum_q h(\hat{\mathbf{A}}_{qj})$.
- During backpropagation, the gradient of the loss E (now w.r.t $\hat{\mathbf{A}}$) is updated using the chain rule as

$$\frac{\partial E}{\partial \hat{\mathbf{A}}_{ij}} = \sum_p \frac{\partial E}{\partial \mathbf{A}_{pj}} . \frac{\partial \mathbf{A}_{pj}}{\partial \hat{\mathbf{A}}_{ij}} \quad \text{with} \quad \frac{\partial \mathbf{A}_{pj}}{\partial \hat{\mathbf{A}}_{ij}} = \frac{h'(\hat{\mathbf{A}}_{ij})}{\sum_q h(\hat{\mathbf{A}}_{qj})} . (\delta_{pi} - \mathbf{A}_{pj}).$$

- In practice $h(.) = \exp(.)$ and the new gradient (w.r.t $\hat{\mathbf{A}}$) is obtained by multiplying the original one by the Jacobian $\mathbf{J}_{\text{stc}} = \left[\frac{\partial \mathbf{A}_{pj}}{\partial \hat{\mathbf{A}}_{ij}}\right]_{p,i=1}^{n}$ which merely reduces to $[\mathbf{A}_{ij} . (\delta_{pi} - \mathbf{A}_{pj})]_{p,i}$.

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Orthogonality

- Learning multiple $\{\mathbf{A}_k\}_k$ allows us to capture different graph topologies when achieving aggregation and convolution.

$$(\mathcal{G} \star \mathcal{F})_{\mathcal{V}} = f\left(\sum_{k=1}^{K} \mathbf{A}_k \mathbf{U}^\top \mathbf{W}_k\right)$$

$$\begin{array}{ll} \min_{\{\mathbf{A}_k\}_k, \mathbf{W}} & E\big(\mathbf{A}_1, \ldots, \mathbf{A}_K; \mathbf{W}\big) \\ \text{s.t.} & \mathbf{A}_k \odot \mathbf{A}_k > \mathbf{0}_n, \ \mathbf{A}_k \odot \mathbf{A}_{k'} = \mathbf{0}_n \quad \forall k, k' \neq k. \end{array}$$

- We consider $\exp(\gamma \hat{\mathbf{A}}_k) \oslash (\sum_{r=1}^{K} \exp(\gamma \hat{\mathbf{A}}_r))$ as a softmax reparametrization of $\mathbf{A}_k$, with $\{\hat{\mathbf{A}}_k\}_k$ free parameters in $\mathbb{R}^{n \times n}$.
- By choosing a large value of γ, it becomes possible to implement ϵ-orthogonality ; a surrogate property where only one entry $\mathbf{A}_{kij} \gg 0$ while all others $\{\mathbf{A}_{k'ij}\}_{k' \neq k}$ vanish.
- The setting of γ and updated Jacobians are in the paper.

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Symmetry and combination

- Symmetry is guaranteed by considering the reparametrization of each matrix as $\mathbf{A}_k = \frac{1}{2}(\hat{\mathbf{A}}_k + \hat{\mathbf{A}}_k^\top)$ with $\hat{\mathbf{A}}_k$ being free.
- Symmetry is maintained by multiplying the original gradient $\frac{\partial E}{\partial \mathbf{vec}(\{\mathbf{A}_k\}_k)}$ by the Jacobian

$$\mathbf{J}_{\text{sym}} = \frac{1}{2}\left[1_{\{k=k'\}} \cdot 1_{\{(i=i',j=j')\vee(i=j',j=i')\}}\right]_{ijk,i'j'k'}.$$

- One may combine symmetry with all the aforementioned constraints by multiplying the underlying Jacobians, so the final gradient is obtained by multiplying the original one as

$$\frac{\partial E}{\partial \mathbf{vec}(\{\hat{\mathbf{A}}_k\}_k)} = \mathbf{J}_{(\text{sym or stc})} \cdot \mathbf{J}_{\text{orth}} \cdot \frac{\partial E}{\partial \mathbf{vec}(\{\mathbf{A}_k\}_k)}.$$

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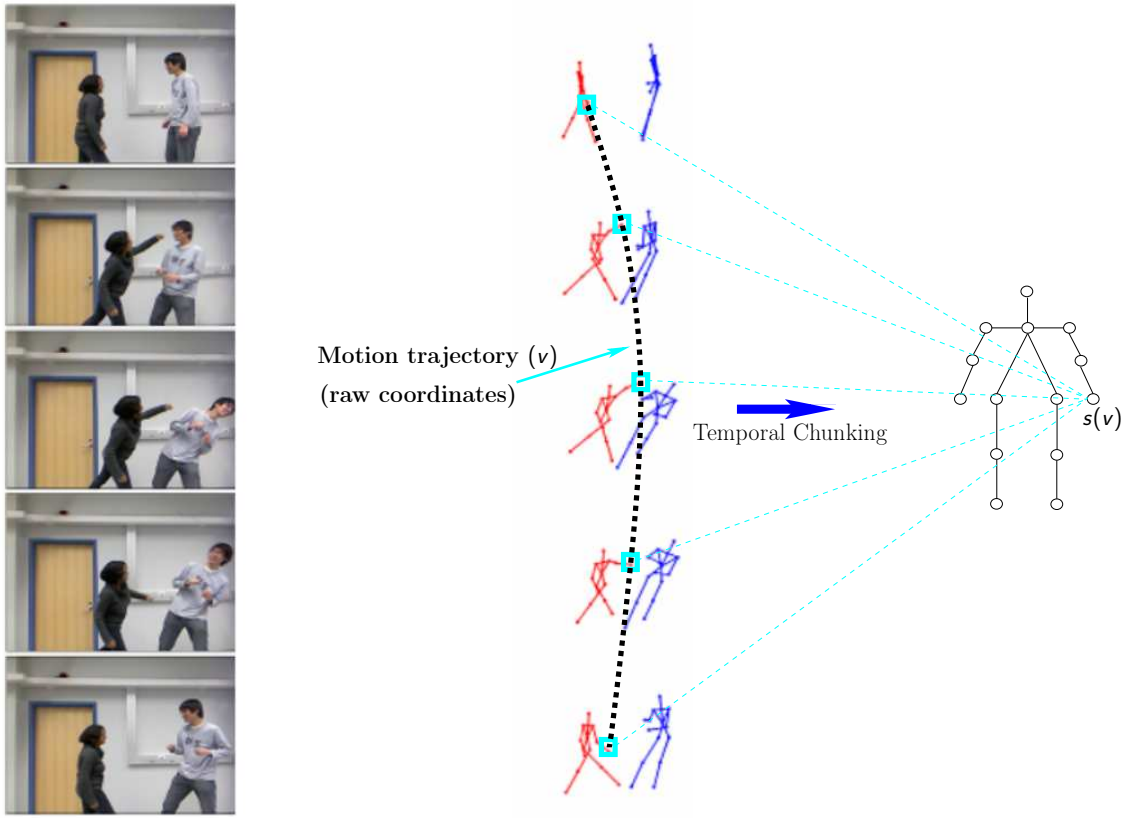
Outline

Database and settings

- We evaluate our GCN on the task of action recognition, using the SBU Kinect dataset.
- This is an interaction dataset acquired using the Microsoft Kinect sensor ; it includes in total 282 video sequences belonging to $C = 8$ categories with variable duration, viewpoint changes and interacting individuals.
- In all these experiments, we use the same evaluation protocol as the one suggested in (SBU12) (i.e., train-test split) and we report the average accuracy over all the classes of actions.
- We trained our GCNs for 3000 epochs, with a batch size of 200, a momentum of 0.9 and a learning rate ν that decreases as $\nu \leftarrow \nu \times 0.99$ (resp. increases as $\nu \leftarrow \nu/0.99$).

Input skeleton graphs

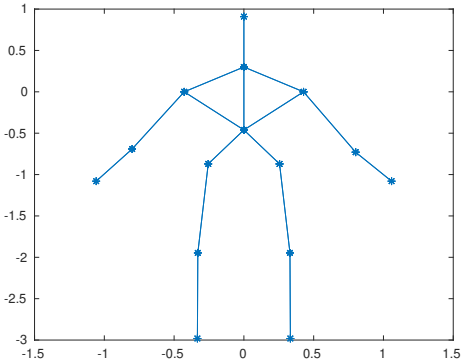


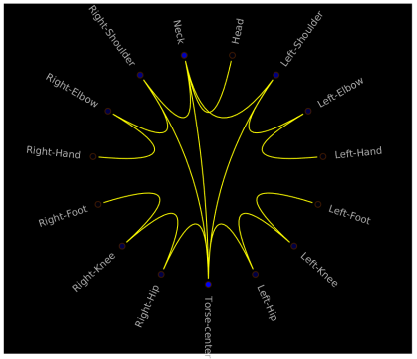
Performances

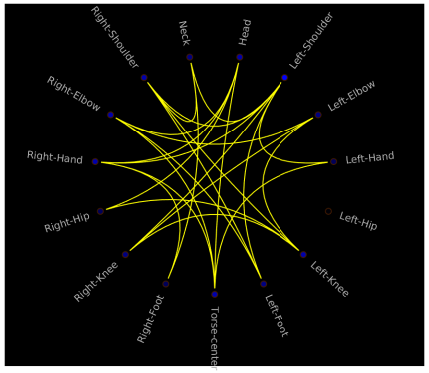
Oper \ Const		none	sym	orth	stc	sym+orth	orth+stc	Mean
HPM.	$K = 1$	89.2308	92.3077	–	89.2308	–	–	90.2564
	$K = 4$	87.6923	89.2308	89.2308	87.6923	90.7692	92.3077	89.4872
	$K = 8$	90.7692	95.3846	92.3077	90.7692	92.3077	92.3077	92.3077
	Mean	89.2308	92.3077	90.7692	89.2308	91.5384	92.3077	90.7692
LPM.	$K = 1$	92.3077	87.6923	–	95.3846	–	–	91.7949
	$K = 4$	92.3077	92.3077	93.8462	95.3846	90.7692	96.9231	93.5897
	$K = 8$	95.3846	90.7692	87.6923	93.8462	93.8462	92.3077	92.3077
	Mean	93.3333	90.2564	90.7692	94.8718	92.3077	94.6154	92.7180
Our	$K = 1$	95.3846	93.8462	–	95.3846	–	–	94.8718
	$K = 4$	93.8462	95.3846	95.3846	96.9231	93.8462	**98.4615**	95.6410
	$K = 8$	92.3077	93.8462	95.3846	90.7692	95.3846	90.7692	93.0769
	Mean	93.8462	94.3590	95.3846	94.3590	94.6154	94.6154	94.4615

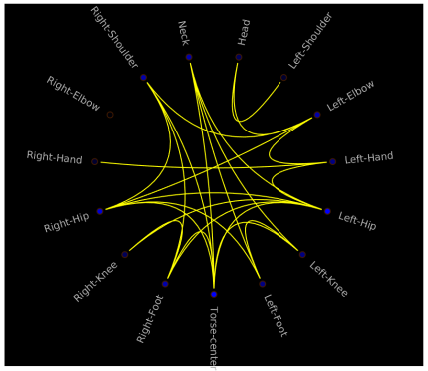
HPM : handcrafted power map connectivity, LPM : learned power map connectivity, sym : symmetry, orth : orthogonality, stc : stochasticity.

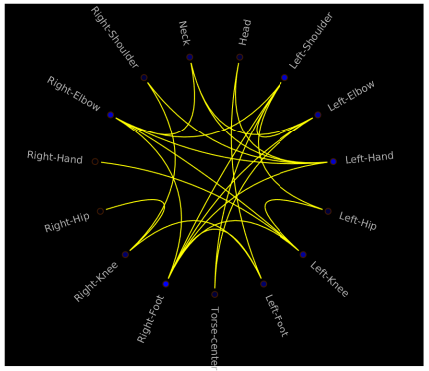
Learned connectivity (examples)

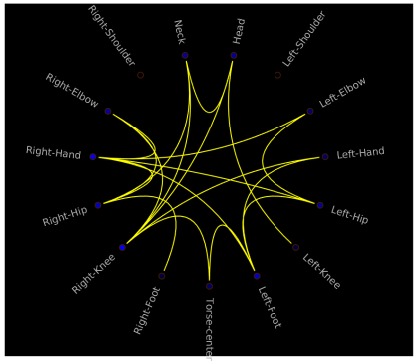












Comparison

Methods	Perfs
GCNConv [57]	90.00
ArmaConv [61]	96.00
SGCConv [59]	94.00
ChebyNet [58]	96.00
Raw coordinates [53]	49.7
Joint features [53]	80.3
Interact Pose [62]	86.9
CHARM [63]	83.9
HBRNN-L [64]	80.35
Co-occurrence LSTM [66]	90.41
ST-LSTM [67]	93.3
Topological pose ordering[70]	90.5
STA-LSTM [56]	91.51
GCA-LSTM [55]	94.9
VA-LSTM [68]	97.2
DeepGRU [54]	95.7
Riemannian manifold trajectory[69]	93.7
Our best GCN model	**98.46**

Outline

Conclusion

- We introduce in this paper a novel method which learns connectivity that "optimally" defines the support of aggregations and convolutions in GCNs.
- We investigate different settings which allow extracting non-differential and differential features as well as their combination before applying convolutions.
- We also consider different constraints (including orthogonality and stochasticity) which act as regularizers on the learned matrix operators.
- Experiments conducted on the challenging task of skeleton-based action recognition show the clear gain of the proposed method w.r.t different baselines as well as the related work.