ECOLE NATIONALE DES SCIENCES DE L'INFORMAUIQUE

Groupe de Recherche en Image et Forme de Tunisie

Fast blending of planar shapes based on invariant invertible and stable descriptors

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Introduction

<u>Shape blending :</u>

❖ building coherent <u>intermediate shapes</u> between a <u>target</u> shape and a <u>source</u> one.

→ Estimating a <u>progressive</u> and <u>continuous</u> transformation.

<u>Focus on Closed Planar Curves Blending :</u>

➢ *linear* approach
➢ *Star-skeleton* approach [4]
➢ *interior polygons* approach [1]
➢ Shape space approach [5]
➢ *curvature signature* approach [3, 6, 12]

Example of blending curves :

Source curve

Target curve

Intermediate generated curves

Linear approach : Registration step + vertices interpolation

Star-Skeleton approach : Registration step + Skeletons interpolation + Reconstruction

Interior polygon approach : Registration step + Polygons interpolation + Reconstruction

Shape Space approach : Registration step + interpolation between elements of shape space + Reconstruction → ***Open curves***.

Curvature signature approach : Registration step + curvatures interpolation + Reconstruction → ***Open curves***.

FOURIER-BASED INVARIANT DESCRIPTOR (FID) BLENDING APPROACH

- Generalization of Surazhsky and al. [6] work & emply FID instead curvature
- FID [8] is complete, invertible, stable [7] and invariant under SE(2).
- Globally description.
- Invariant to : *Starting point* & *Euclidean transformations*
- Generates intermediate closed curves
- No Registration step
- No Post-processing (*B-spline*)
- Computationally-efficient (*O(nlog(n))*)

GENERALIZATION OF SURAZHSKY AND AL. WORK:

Shape Space

Input : Source curve C, Target curve C'

$g(C)$

$g(C')$

Invariant Space

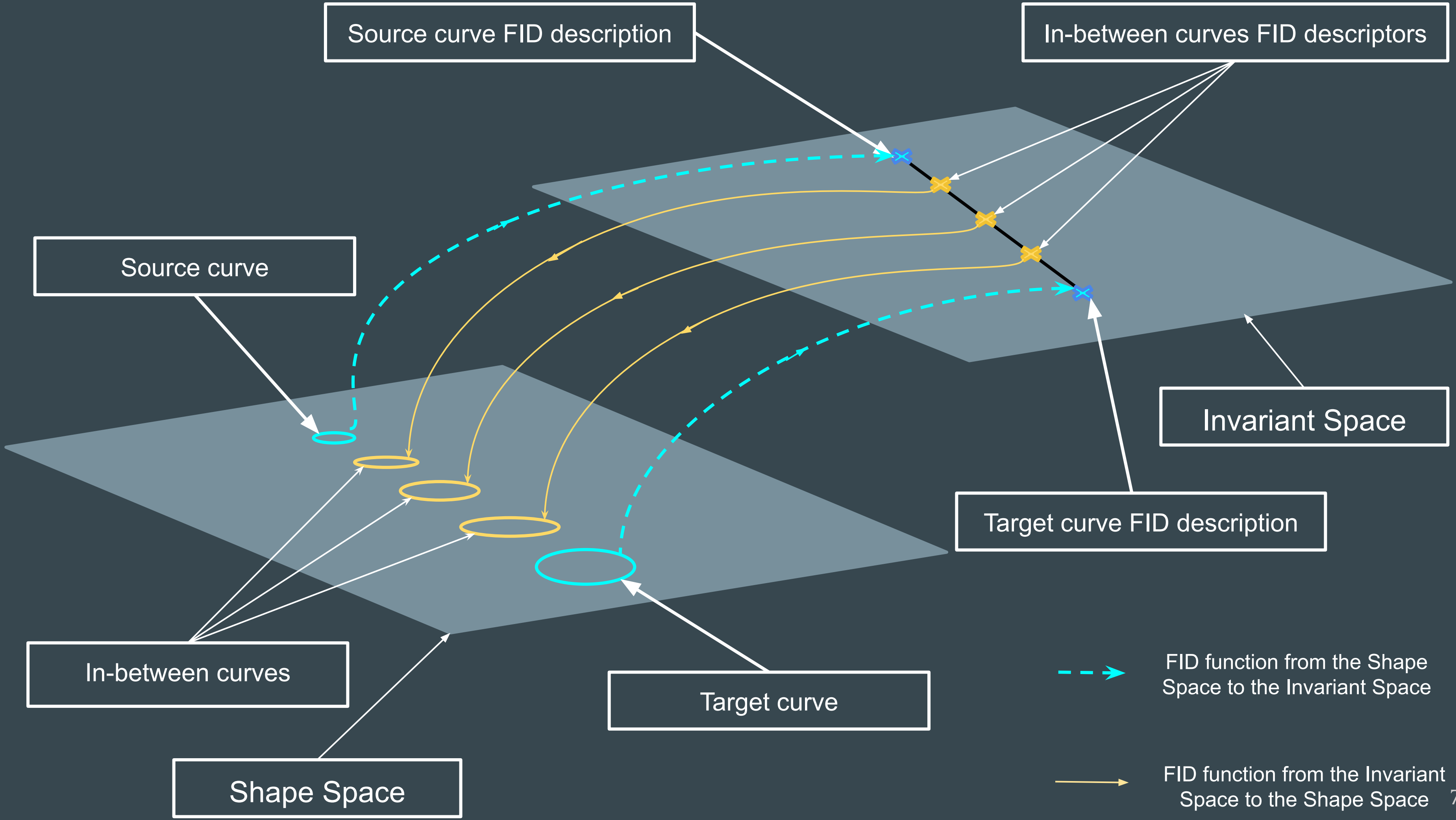
Description of the source curve C

Description of the target curve C'

Interpolation between the target and the source description: $(1\text{-}t)\ g(C)+t\ g(C')$

Shape Space

Reconstruction of the weighted average descriptors through the inverse formula: $f(t)=g\text{-}1((1\text{-}t)\ g(C)+t\ g(C'))$



Source curve FID description
In-between curves FID descriptors
Source curve
Invariant Space
Target curve FID description
In-between curves
Target curve
Shape Space
FID function from the Shape Space to the Invariant Space
FID function from the Invariant Space to the Shape Space

FID Formula and the reconstruction expression

Fourier coefficient of C

$$\begin{cases} I_n = \dfrac{a_n^{n_0-n_1}\, a_{n_0}^{n_1-n}\, a_{n_1}^{n-n_0}}{|a_{n_0}|^{n_1-n-p}\,|a_{n_1}|^{n-n_0-q}}, & \forall n \neq n_0, n_1 \\ I_{n_0} = |a_{n_0}|^{n_0-n_1+p}, & n = n_0 \quad and \quad a_{n_0} \neq 0 \\ I_{n_1} = |a_{n_1}|^{n_1-n_0+q}, & n = n_1 \neq n_0 \quad and \quad a_{n_1} \neq 0 \end{cases}$$

$$a_n = I_n^{\frac{1}{m_0}}\, I_{n_0}^{\frac{-p}{\alpha_1}}\, I_{n_1}^{\frac{-q}{\alpha_2}}\, e^{\frac{2i\pi}{m_0}(n(\theta_0-\theta_1)-n_1\theta_0+n_0\theta_1)}$$

Fourier expansion of C

$$\begin{aligned} m_0 &= n_0 - n_1 \\ \alpha_1 &= m_0(n_0 - n_1 + p) \\ \alpha_2 &= m_0(n_1 - n_0 + q) \end{aligned}$$

θ_0 θ_1 the arguments of a_{n_0} and a_{n_1}

Numerical considerations

A discrete modelization termed fined-FID is introduced from the FID.

Fourier coefficients → **DFT**

Fourier expansion → **IDFT**

Discret Fourier Transform (DFT)

↓

$$\begin{cases} J_n = \dfrac{F_n^{n_0-n_1} F_{n_0}^{n_1-n} F_{n_1}^{n-n_0}}{|F_{n_0}|^{n_1-n-p} |F_{n_1}|^{n-n_0-q}}, & \forall n \neq n_0, n_1 \\ J_{n_0} = |F_{n_0}|^{n_0-n_1+p}, & n = n_0 \quad and \quad F_{n_0} \neq 0 \\ J_{n_1} = |F_{n_1}|^{n_1-n_0+q}, & n = n_1 \neq n_0 \quad and \quad F_{n_1} \neq 0 \end{cases}$$

↓

$$F_n = J_n^{\frac{1}{m_0}} J_{n_0}^{\frac{-p}{\alpha_1}} J_{n_1}^{\frac{-q}{\alpha_2}} e^{\frac{2i\pi}{m_0}(n(\theta_0-\theta_1)-n_1\theta_0+n_0\theta_1)}$$

↓

Inverse Discret Fourier Transform (IDFT)

$$\begin{aligned} m_0 &= n_0 - n_1 \\ \alpha_1 &= m_0(n_0 - n_1 + p) \\ \alpha_2 &= m_0(n_1 - n_0 + q) \end{aligned}$$

θ_0 θ_1 the arguments of a_{n_0} and a_{n_1}

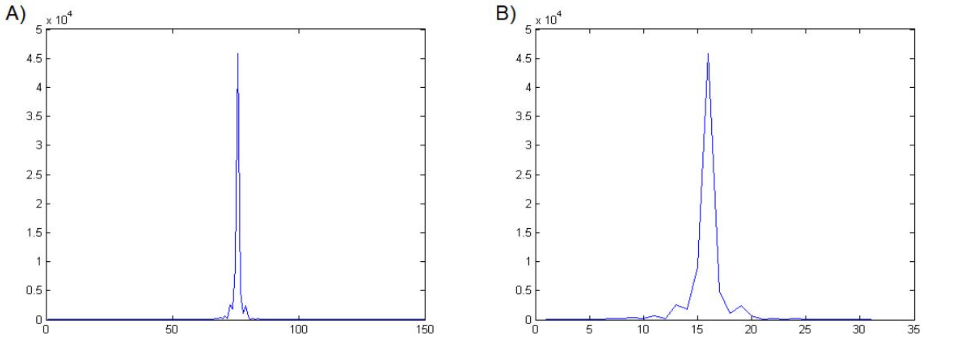
Numerical considerations

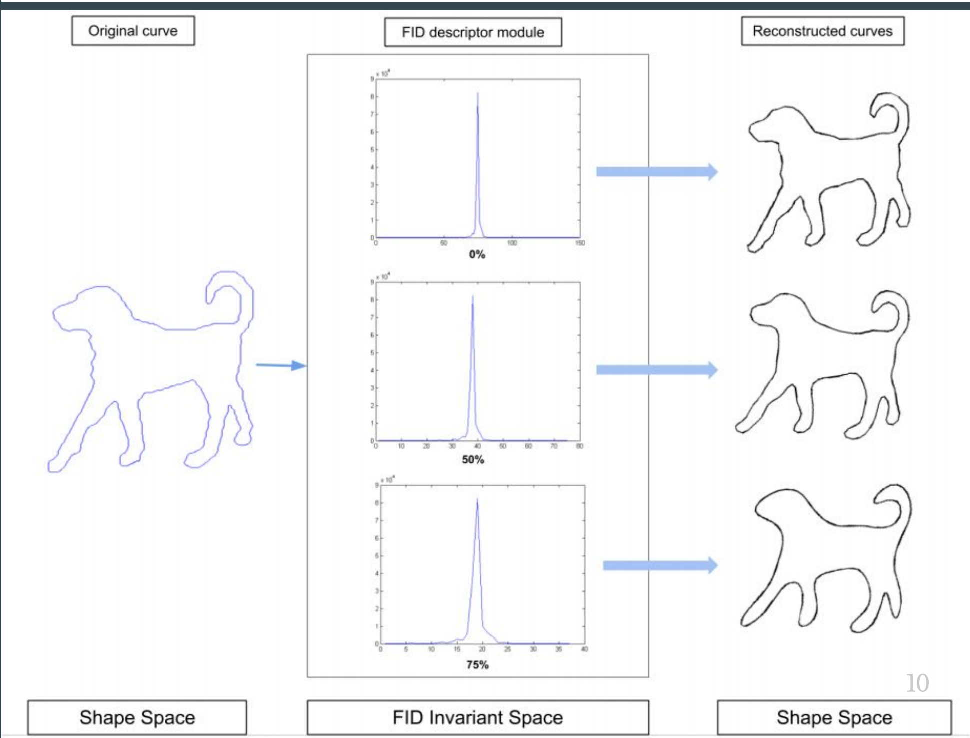
Truncation step → necessary for avoiding numerical complications (infinite values, noise, etc.)

we consider a window around the highest DFT energy coefficients then remove the energy outside.

Window: [n0 – N/10, n0 + N/10], N = 150.

N : the number of shape coefficients.
n0: the highest DFT energy coefficient index.

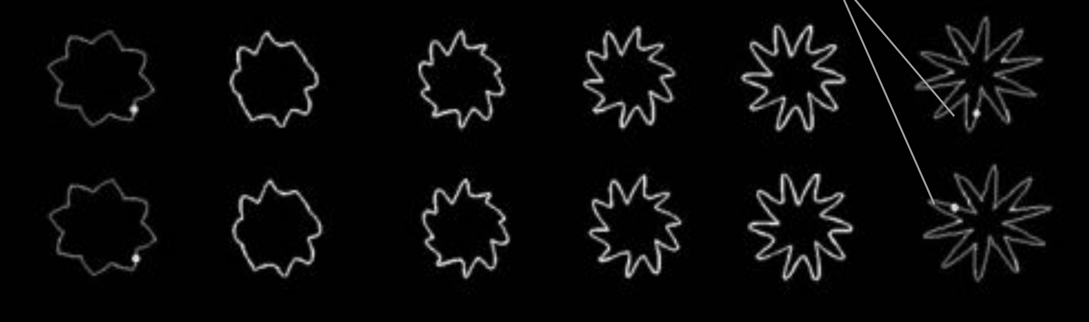


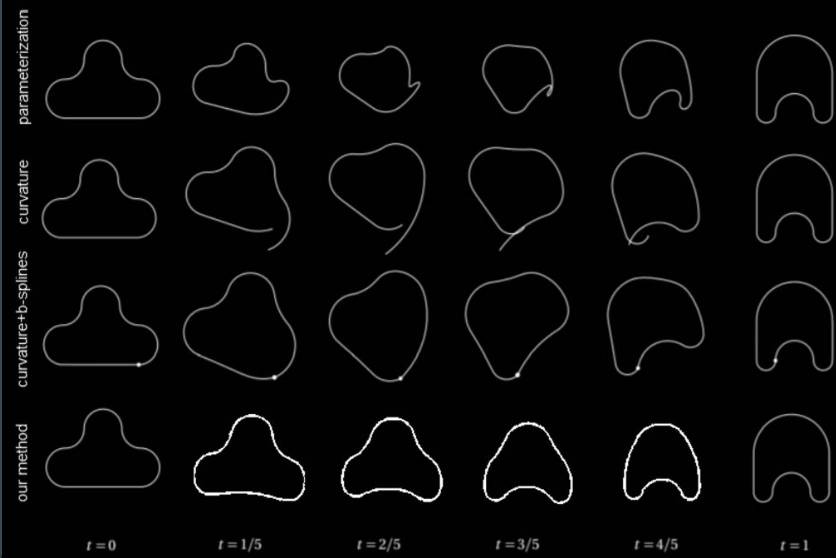


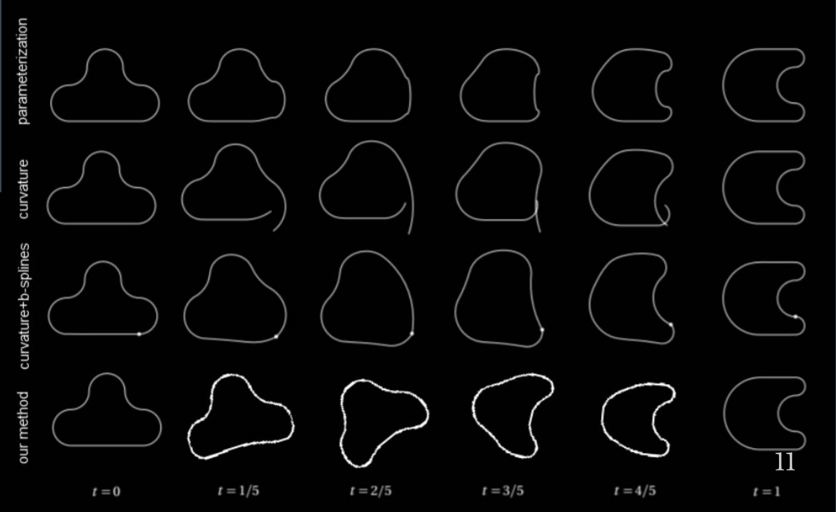
Experiments

- ❖ Comparison between *linear*, *curvature*, *curvature + B-splines* and *fined-FID* approaches.
- ➔ Invariance to Euclidean Transformations & Starting point
- ➔ No registration needed !
- ➔ Generates closed curves !

Different Starting points



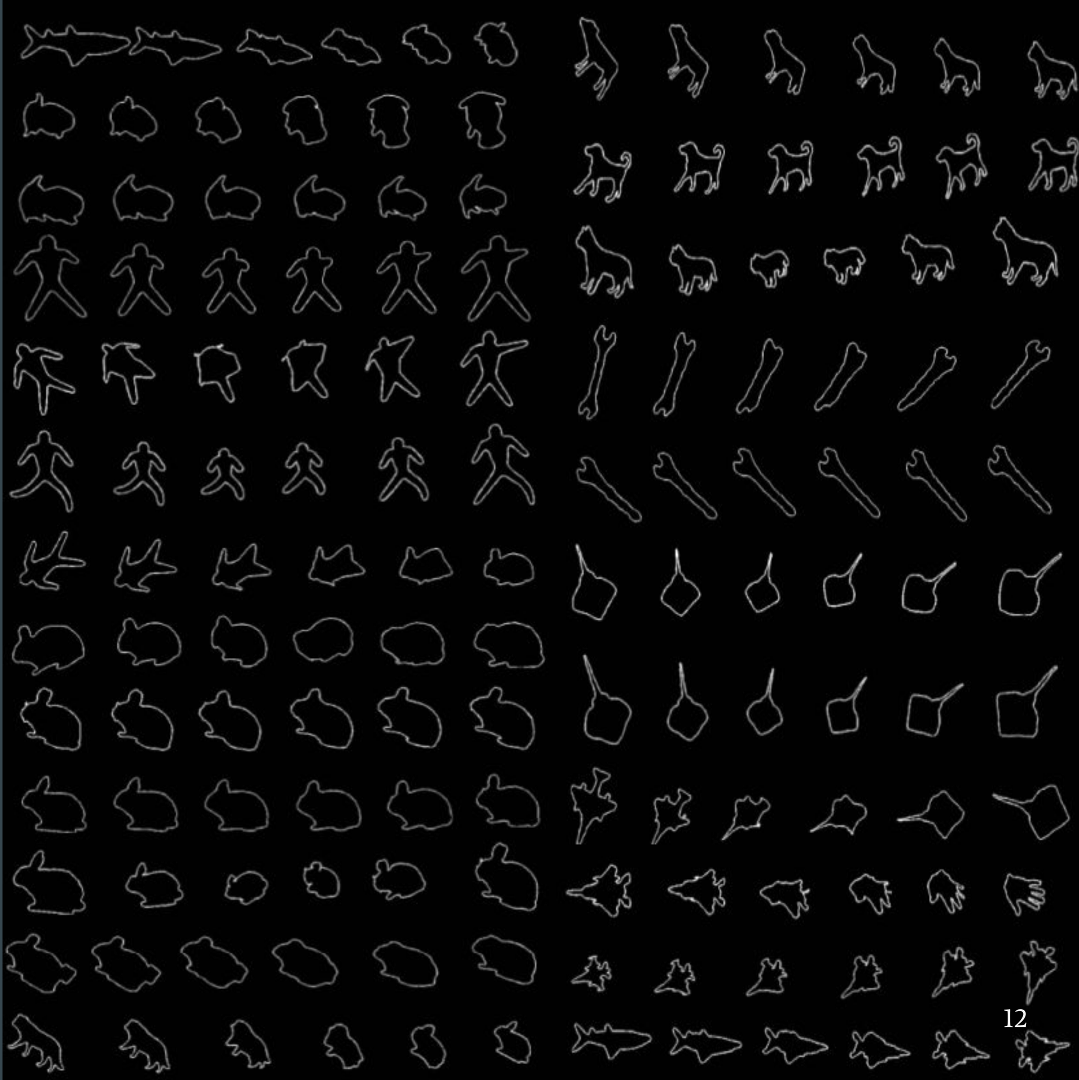




Fined-FID blending approach on KIMIA99 Database

Observations on experiments:
Two cases:
(1) two curves belong to different classes.
(2) target and source curves belong to the same class

Remarks with visual criterion:
(1) No Registration needed for the first case → more good results.
(2) Registration step is in general necessary to generates meaningful in-between curves → less good results in this case.



Conclusion & Perspectives

- General approach for the interpolation of continuous and closed curves based on invariant descriptors is introduced
- Fined-FID is introduced which is a discrete modelization of the FID
- Considering the interpolation of open curves
- Carry out an optimization study to find the best (non-linear) path in the invariant space

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