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Generalized conics: properties and applications

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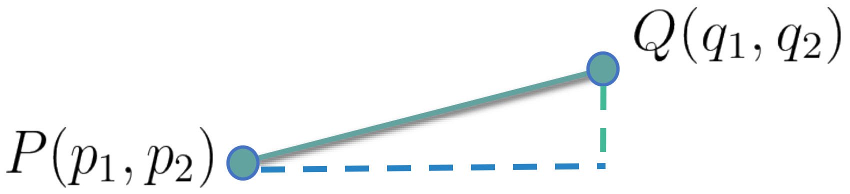
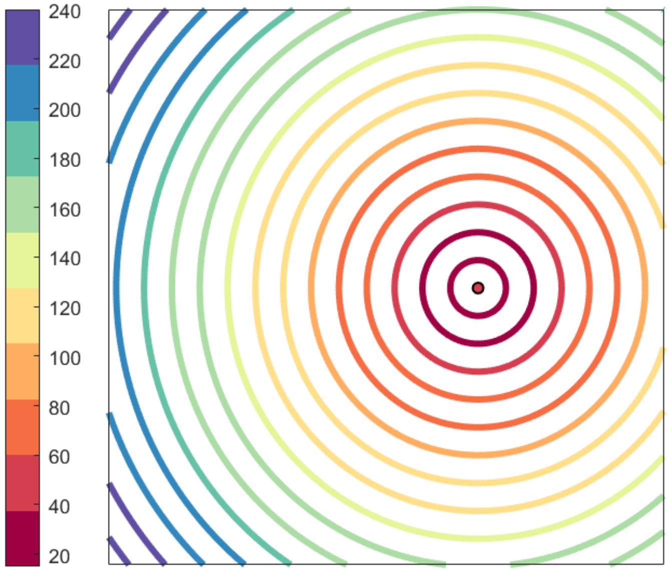
Distance between the objects

... is a numerical measure of how close two objects are to each other

Example: Euclidean distance in 2D



$$\delta(P, Q) = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2}$$



Euclidean distance field (EDF) of the point contains concentric circles

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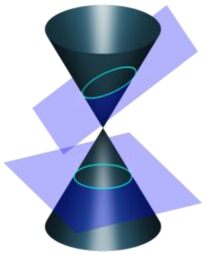
Distance field from Ellipses

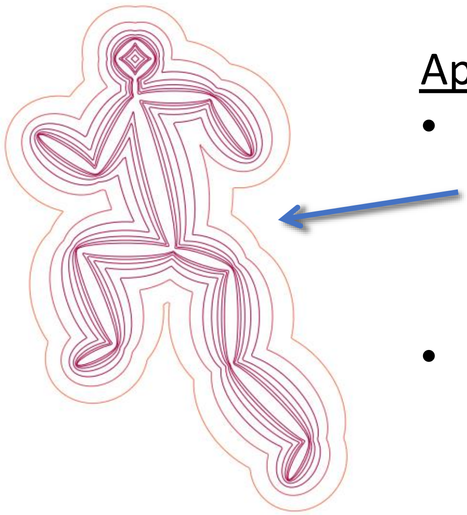


https://en.wikipedia.org/wiki/Conic_section

Ellipse is the locus of points for which the sum of the distances to two focal points is constant

Distance Field of confocal ellipses (CE)

- **pixel-wise sum** of the Euclidean distance fields
- computes the distance from point to line segment using only the endpoints
- invariant to rotation, translation, and discretization of the line segment



Applications:

- distance field of a shape by taking a **pixel-wise minimum** of several CEs: **Confocal Elliptic Field (CEF)** [1]
- elliptic representation of the rigid parts of the shape

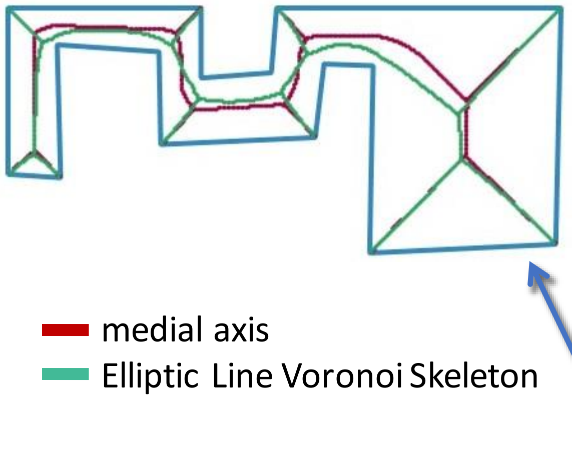
Distance field from Hyperbolas

Hyperbola is the locus of points with constant absolute difference between the distances to two focal points

https://en.wikipedia.org/wiki/Conic_section

Distance Field of confocal hyperbolas (CH)

- **pixel-wise difference** between the Euclidean distance fields
- tessellates the space: proximity to the focal point w.r.t. the sign of the distance value



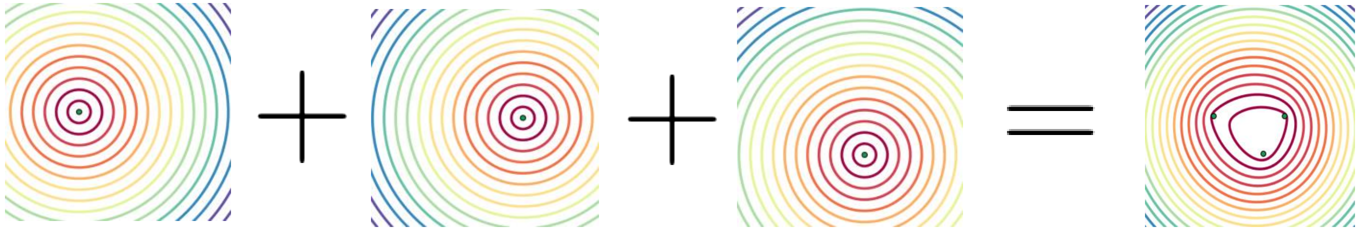
Applications:

- Elliptic Line Voronoi Diagram and Skeleton [2] – pixel-wise difference of multiple CEFs: **Confocal Hyperbolic Field(CHF)**
- shape smoothing [2]
- optimal route planning

Distance field from Generalized Conics-1

Generalized conic is a geometrical object defined by a property which is a generalization of some defining property of the classical conic

Multifocal ellipse is a locus of points for which the sum of the weighted distances to N focal points is constant

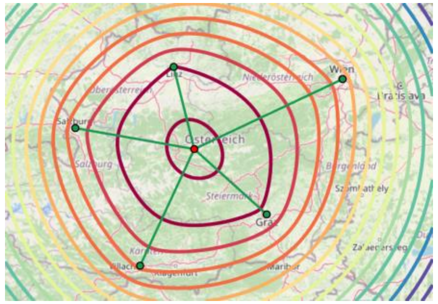


Confocal multifocal elliptic field (CMEF)

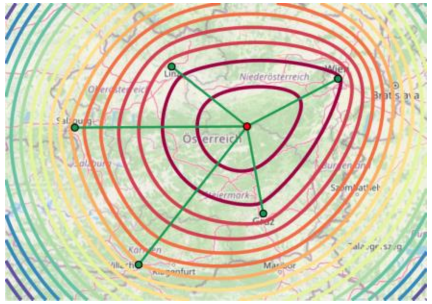
- **pixel-wise sum** of the Euclidean distance fields
- level sets are convex
- single global minimum except for the even number of collinear focal points
- invariant to rotation, translation, and scaling

<u>Application:</u>

- Fermat-Torricelli and Weber problem [3]



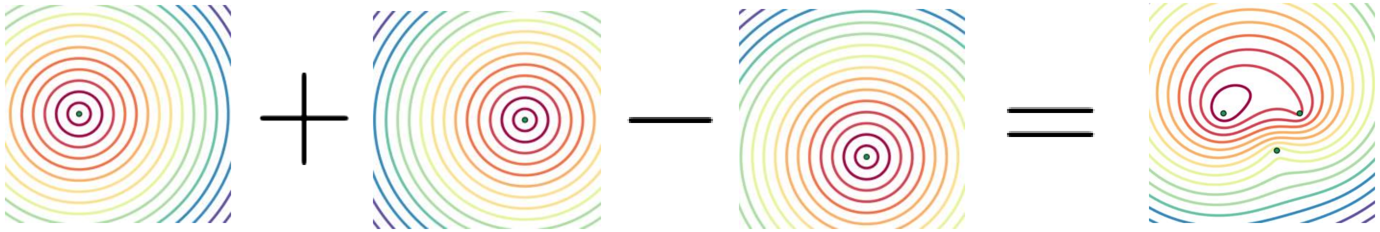
equal weights



non-equal weights

Distance field from Generalized Conics-2

Multifocal hyperbola is a locus of points with a constant absolute difference between two multifocal ellipses

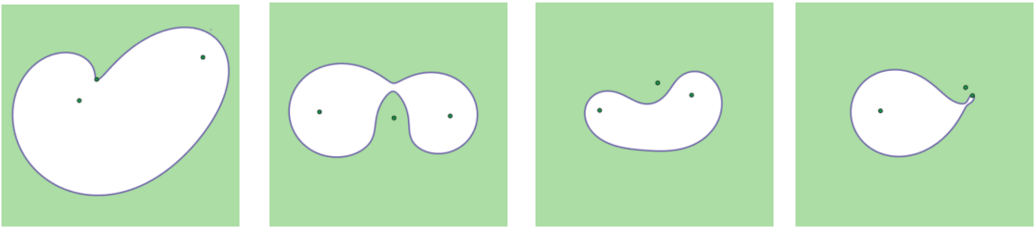


Confocal multifocal hyperbolic field (CMHF)

- **pixel-wise sum** followed by **pixel-wise difference** of the Euclidean distance fields
- level sets are not necessarily convex
- generates concavities

Application:

- has a potential for compact shape representation

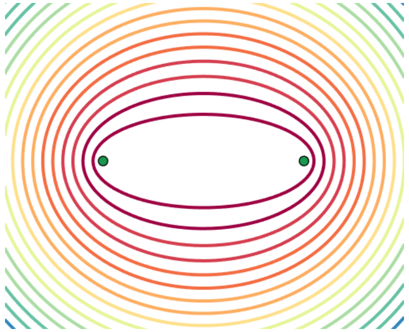


various shapes generated from three focal points with positive and negative weights

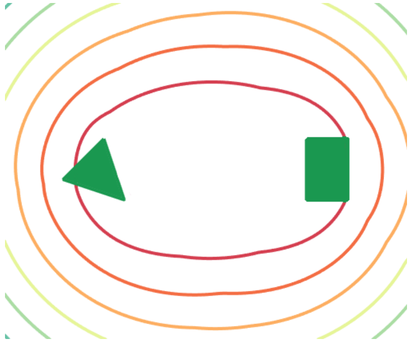
An extended view on focal points

- **can be represented by complex objects**

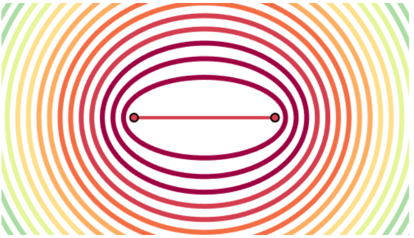
multifocal ellipse, where the focal points are represented by the points



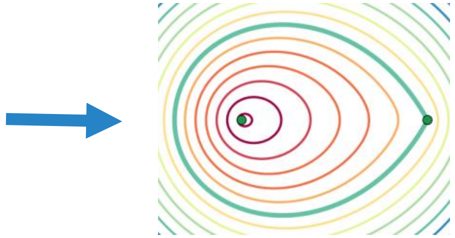
multifocal ellipse, where the focal points are represented by the triangle and rectangle



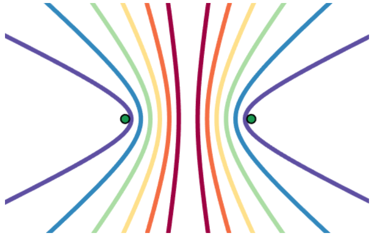
- **can have weights**



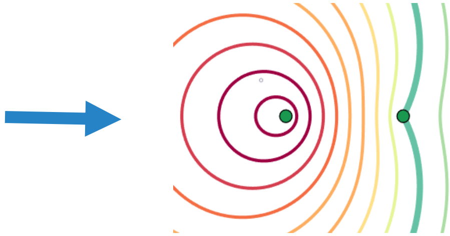
CE with equal weights



CE with non-equal weights



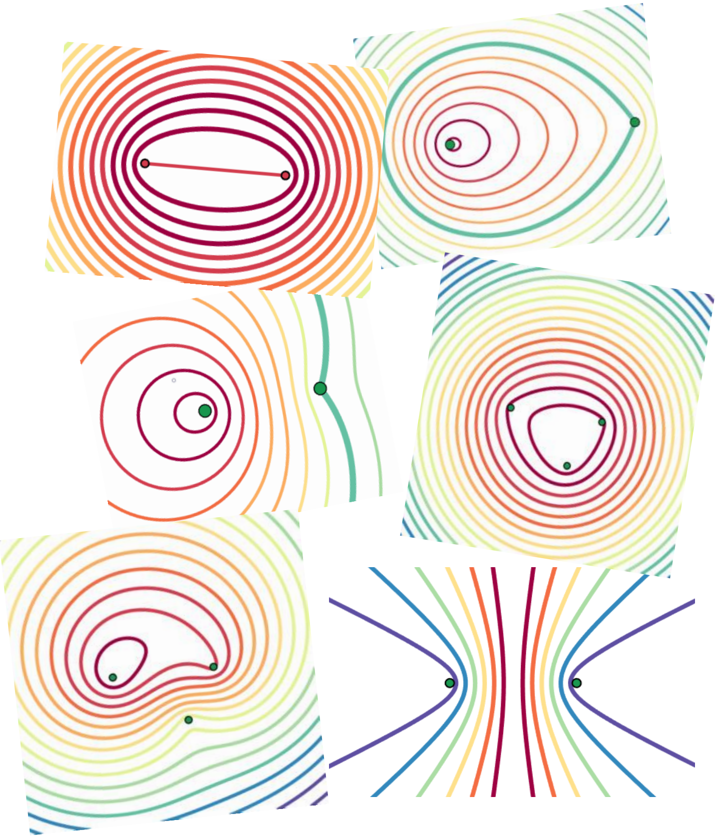
CH with equal weights



CH with non-equal weights

Summary

- Simple pixel-wise operations generate new types of distance fields
- Distance field of confocal ellipses (CE) : requires only two end points of the line segment [1]
- Confocal Elliptic Field (CEF) : distance field of a shape [1,2]
- Confocal multifocal elliptic field (CMEF) : total sum of the distances to the given focal points
- Confocal multifocal hyperbolic field (CMHF) : the tessellation of the space with regard to some metric
- Hierarchical representations : by combining and weighting multiple distance fields



References

[1] A. Gabdulkhakova and W. G. Kropatsch, “Confocal ellipse-based distance and confocal elliptical field for polygonal shapes,” in 24th International Conference on Pattern Recognition, 2018, pp. 3025–3030.

[2] A. Gabdulkhakova, M. Langer, B. W. Langer, and W. G. Kropatsch, "Line Voronoi Diagrams using elliptical distances", In: *Joint IAPR International Workshops on Statistical Techniques in Pattern Recognition (SPR) and Structural and Syntactic Pattern Recognition (SSPR)*. 2018. pp. 258-267.

[3] C. Groß, T. K. Strempel, "On generalizations of conics and on a generalization of the Fermat-Torricelli problem", *The American mathematical monthly*, *105*(8), 732-743.